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## USER'S GUIDE



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The Semi-analytic Tool for End of Life Analysis software (STELA) is a semi-analytic orbit propagator.

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## 1. STELA purpose

The Semi-analytic Tool for End of Life Analysis software (STELA) has been designed by CNES (The French Space Agency) to support the French Space Act. When the impossibility to carry out a controlled reentry is duly proven, an uncontrolled reentry or a stable disposal orbit can be chosen, given that the orbit respects the different criteria established in the French Space Operations Act. STELA reflects the standard concerning the protection of LEO and GEO regions (lifetime and protected regions crossing of disposal orbits) and provides the user with tools to assess compliance with the requirements. The software allows efficient long-term propagation of LEO, GEO, and GTO types orbits based on semi-analytical models, statistical analysis and assessment of protected regions criteria. STELA produces a report file that summarizes the computation (spacecraft characteristics, initial and final orbits, computation parameters, criteria status) and optionally an ephemeris file. For GTO orbits, due to resonances phenomena, a statistical analysis is performed using Monte-Carlo method.

STELA software includes:

- an iterative computation mode adjusting the initial orbit to achieve a given atmospheric reentry duration, or to avoid GEO region crossing for a given duration,
- a tool that computes the cross sectional mean area of a spacecraft,
- a tool that converts Two Lines Elements into STELA orbital elements.

STELA can also be used in batch mode and as a java library.
STELA contact: stela@cnes.fr

## 2. Getting Started

### 2.1. System Configuration

The Oracle/Sun Java Runtime Environment release 1.8 must be available in the system configuration in order to install and run STELA software.

Java virtual machine needs 512 Mo of RAM. This is defined in files stela.sh, stela.bat, stela-batch.sh and stela-batch.bat in bin directory through the option "-Xmx512m". For specific and advanced use of STELA requiring more memory allocation, you can change this parameter.

To benefit from 3D functionalities of Mean area tool, OpenGL libraries must be available:

- opengl32.dll for Windows
- libGL.so for Linux

STELA has been tested on x86 platforms, with the following operating systems : Windows 10, Linux Red Hat 764 bits.

The STELA software can run with JRE 32 bits or JRE 64 bits. In all cases, the platform and the Java Runtime Environment release should be consistent. We recommend the use of Linux 64 bits for execution time. On the same machine, the computation time is about 5\% lower with JRE 64 bits than with JRE 32 bits.

STELA performance can be affected by additional factors, notably by the fact the JRE optimizes itself differently according to the platform it runs on.

- The JRE automatically runs a "client-class" JRE on these environments : Windows 32 bits, Linux and Solaris 32 bits with less than 2GB ram.
- The JRE automatically runs a "server-class" JRE on these environments : Windows 64 bits, Linux and Solaris 32 bits with more than 2GB ram, Linux and Solaris 64 bits.

The server-class JRE takes more time to start, but is faster overall. STELA is better suited for the "server-class" JRE, since its startup time is neglectable compared to its usual processing time.

When Stela starts (batch or GUI mode), server mode is activated if this mode is available.
STELA can be installed for a single user or for a multi-users environment, according to the system rights available for the user. The available system rights vary according to the system credentials : for further details or if installing STELA on a multi-users environment, contact system administrator.

### 2.2. Software Installation and Removal

### 2.2.1. Install STELA

IMPORTANT NOTES :

- Having a proper Oracle/Sun Java Runtime Environment (version 1.8) installed is required to run STELA.
The STELA installer also requires this.
Ideally the latest version of the JRE should be installed to ensure best GUI compatibility.
WARNING: STELA may not work with IBM JRE (or any other JRE than Oracle) and it is strongly recommended not to use them.
- If a new version is installed in the same folder as the previous one, all files will be overwritten. User modified files have to be saved previously.


### 2.2.1.1 Install on Windows

1. Double-click on the setup file "stela-install-[X.X.X].jar", where [X.X.X] is the release of STELA.

The installation program will ask to fill in the following field
2. Information about STELA release are displayed to the user (example below is for version 1.2.0)

3. In order to go through the next step, the user must accept the terms of the license agreement, and click on the button "Next".

Please read the following license agreement carefully:
Please read attentively the provisions of this Licence before downloading the SOFTWARE. The use of the SOFTWARE by the Licensee means the latter has agreed with the provisions of this Licence.

The SOFTWARE as described thereof, property of the CNES and named "STELA" has been registered at the "Agence pour la Protection des Programmes" (119 rue de Flandres, 75019 Paris) on January 2011 the 5th under the number LO1101.
On the SOFTWARE, CNES concedes to the Licensee, who can be a physical or a moral person, a non exclusive free Licence.

LANGUAGE OF THE LICENCE AND APPLICABLE LAK
The Licence is established in French and English. In the case of a dispute, the French version is the one that prevails.
This Licence is governed by French law. If any dispute should arise, litigation shall be brought before the applicable courts.

I accept the terms of this license agreement.

- I do not accept the terms of this license agreement
(Made with IzPack - http://izp ack.org/)


4. Installation path selection : a new window appears with a browser. The default directory already appears in the installation path. The "Browse" button enables the user to choose another location in which STELA is to be installed.

If the chosen directory does not exist, a pop-up will ask the user to confirm or cancel the creation of the target directory.


Click OK to continue.
If the directory exists, a pop-up will ask the user to erase the existing version of software. (see § Install a new version of STELA)


Click Yes to continue.
5. Packs Selection : a new window appears in order to allow the user to choose the packages he wants to install (example below is for version 1.2.0).


Clicking on＂next＂will proceed to the software installation．

6．Before exiting the installation program，the user can set＂STELA＂shortcuts by clicking on＂Next＂． The delivery contains an icon＂bin／stela．ico＂that will be used for the shortcut．

7. The user can add the STELA program in the Windows start menu by selecting "Create shortcuts in the Start-Menu" or can set a shortcut on the desktop with "Create additional shortcuts on the desktop".

## IzPack - Installation of STELA

$\square$

## Setup Shortcuts

Create shortcuts in the Start-Menu
Greate additional shortcuts on the desktop
Select a Program Group for the Shortcuts:

```
Accessoires
CollabNet Subversion Client
Gygwin
Cygwin-X
Démarrage
Enterprise Architect }
FreeMind
GIMP
InfraRecorder
Jeux
```

STELA

(Made with IzPack - http://izpack.orgh)
8. The installation is complete when the following window appears :

政 Installation has completed successfully.
${ }_{6}^{\Omega}$ C: \Program Files \STELA'U_Uninstaller

### 2.2.1.2 Install on Linux

In a shell, run the installer: java -jar stela-install-[X.X.X].jar, where [X.X.X] defines the release of STELA.

For next steps, refer to the Installation on Windows.

### 2.2.1.3 Install on Sun-Solaris

In a shell, run the installer: java -jar stela-install-[X.X.X].jar, where [X.X.X] defines the release of STELA.

For next steps, refer to the Installation on Windows.

### 2.2.2. Uninstall STELA

The user can uninstall the product STELA by using the Windows menu "Démarrer" ("Start") :


The following window will appear :


If the user clicks on "Uninstall", the STELA software will be removed.

### 2.2.3. Install a new version of STELA over an existing one

To install a new version of Stela software, follow procedure below :

- Copy the modified configuration files (e.g.: configuration/stela_drag_coefficient) of the already installed version in a backup folder.
- Copy simulation files (e.g.: example/leo_sim_2011_05_03_SMOS_sim.xml) of the already installed version in a backup folder.
- Uninstall STELA
- Install the new version of STELA
- Merge the old configuration files with the new ones (the structure of the configuration files may have changed between the previous and new version).
- Copy simulation files in the new example folder (don't erase existing simulation example file)


### 2.3. Installation directory

The contents of the installation directory are as follows :

| Name | File/directory | Description |
| :--- | :---: | :---: | :---: |
| Readme.txt | File | Contains the STELA version |
| License.txt, <br> License_FR.pdf | Files | The CNES licence in text and <br> PDF format |
| bin | Directory | STELA launchers and icons |
| configuration | Directory | Contains the configuration files <br> for STELA that may be <br> modified by advanced users <br> (don't forget to comment <br> modifications) |
| Stela-User-Manual.pdf | File | Contains the user manual (pdf <br> format) |
| examples | Directory | Contains example files that can <br> be opened by STELA <br> *_sim.xml) or by the mean area <br> tool (_*shap.xml) |
| (ib | Directory | Contains all java .jar files for <br> STELA and its dependencies |
| resources | Directory | STELA that are not meant to be <br> modified, even by advanced <br> users |
| installationInformation | File | Needed for the uninstaller tool |
| Uninstaller | Directory | Contains the uninstaller tool |

## 3. Using STELA

### 3.1. Run STELA software

Please note that several instances of STELA can run concurrently at the same time, even from the same installation directory. Each instance is fully independent and uses its own system resources (PID, memory...).

However STELA simulation or tool files are not protected from damage if one is accessed by several STELA instances at the same time.

### 3.1.1. GUI mode

Be careful : In GUI mode, input parameters of simulation are rounded before extrapolating, in order to respect the number of digits displayed. This rounding can produce differences between GUI mode and Library or Batch mode.

### 3.1.1.1. Run STELA on Windows

Run the command file "stela.bat" located in the "bin" subdirectory of STELA installation path, or double-click the STELA shortcut of your desktop.

### 3.1.1.2. Run STELA on Linux

Run the shell "stela.sh" located in the "bin" subdirectory of STELA installation path.

### 3.1.1.3. Run STELA on Sun-Solaris

Run the shell "stela.sh" located in the "bin" subdirectory of STELA installation path.

### 3.1.2. Batch mode

### 3.1.2.1. Run STELA on Windows

Run the shell "stela-batch.bat" located in the "bin" subdirectory of STELA installation path.

### 3.1.2.2. Run STELA on Linux

Run the shell "stela-batch.sh" located in the "bin" subdirectory of STELA installation path. Use option "--help" to read documentation.

A shell can be created to automate process. Two examples of script are provided in the «example» folder of STELA installation path (see below 3.1.3.).

The script example_batch.ksh loads an existing simulation file and performs several extrapolations, the second script example_batch.py provides the same computation in Python.

### 3.1.2.3. Run STELA on Sun-Solaris

Run the shell "stela-batch.sh" located in the "bin" subdirectory of STELA installation path.

### 3.1.3. Batch mode examples

Two examples of script are provided in the «example» folder of STELA installation path:

- example_batch.ksh: example in ksh.
- example_batch.py: example in python.

Run them to launch the corresponding script.

### 3.1.3.1. Script ksh (Linux / Sun-Solaris)

The script is divided in two sections : "Methods" and "Main". Only the section "Main" should be modified by the user.

This script simply performs several extrapolations. As an example, The semi-major axis is reduced by 1 km at every iteration.

### 3.1.3.2. Script python (Windows / Linux / Sun-Solaris)

Prior to loading this script, Python has to be installed on the machine. This can be done by downloading Python on the website http://www.python.org.
Warning: this script has been validated only with version 2.7.2. of Python.
The script is divided in two sections : "Methods" and "Main". Only the section "Main" should be modified by the user.

This script simply performs several extrapolations. As an example, the semi-major axis is reduced by 1 km at every iteration.

### 3.1.4. Parallel computing

In order to decrease the computation time for GTO statistical mode, STELA will run the extrapolations in multiprocessing mode (enabled by default). This mode can be disabled in the GUI by unchecking the corresponding checkbox in the Statistics parameters section. For further customization, the number of processes launched may be changed. Parallel computing is used only for GTO statistical mode.

When running STELA on a cluster, one must specify in the batch mode shell the maximum size of used random access memory. Edit the stela-batch.* file and add option of java program : -Xmx512m.

Note: in order to obtain the same statistical result with or without parallel computing, the extrapolation results are taken into account in the ascending order in the statistical analysis. It means that if the result of extrapolation number $k+1$ is available before the result of extrapolation number $k$, the process will wait for the result of extrapolation number $k$ to compute the statistical results at step $k$. As a result, it may lead to unsorted $\log$ messages.

### 3.2. STELA main window features

STELA first proceeds to the opening of the main window as follows (default size is $1024 \times 768$ ):


Functionalities provided by this window are listed below.

### 3.2.1. Create a new simulation

In order to create a new simulation, the user can click on the specific button "New LEO simulation", "New GEO simulation" or "New GTO simulation" or select "New" in the File menu.



### 3.2.2. Save current simulation

In order to save the current simulation, the user can either click on the specific button "Save simulation..." or "Save simulation as...". The user can also select options "Save simulation..." or "Save simulation as..." by the File menu.

or


STELA will save the following files (see §Output data and plots):

- *_sim.xml file that contains the simulation context,
- *_sim.txt file that contains the simulation context and synthesis,
- *_log.txt file that contains the log outputs,
- *_sim_stat.txt file that contains the statistical data and results.


### 3.2.3. Open an existing simulation

An existing simulation can be loaded by clicking on the specific button "Open simulation..." or by selecting "Open simulation..." by the File menu.

or

| New |  |
| :--- | :--- |
| Open simulation... |  |
| Save simulation... |  |
| Save simulation as... |  |
| Exit | $\mathrm{Ctrl}+\mathrm{Q}$ |

### 3.2.4. Run extrapolation

The user can run an extrapolation by clicking on the button "Run extrapolation...".


If no simulation has been opened or created, the buttons "Save simulation", "Save simulation as", and "Run extrapolation" are not available.

### 3.2.5. Tools

The user can select "Tools" in order to start the STELA tool "Mean Area Computation" or the tool "Convert Two-Line elements".


Tools can also be accessed via the toolbar:

|  |  |  |  |  |  |  | STELA |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| File Iools ? |  |  |  |  |  |  |  |
| LEO GEO GTO |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |

### 3.2.6. Help

The user can reach the User Manual of STELA software by selecting the Help option as follows :


### 3.2.7. About STELA

In order to get information about the STELA release and licenses, the user can activate the Help menu.


### 3.2.8. Exit STELA

The user can close the STELA software by selecting "Exit" in the File menu.


### 3.2.9. Logbook

At the bottom of the main window, a logbook contains the history of the user handlings. The logbook also contains STELA warning or error message in case of bad parameters input or extrapolation error.

| Log | Message | Level |
| :---: | :---: | :---: |
| Date |  | (S) INFO |
| $04 / 11 / 201111: 43: 01$ | New LEO simulation created |  |

### 3.2.10. Tooltips

Help tooltips appear when the mouse pointer is over an input parameter name.



### 3.2.11. Progress bar

A progress bar is displayed when STELA is computing (whether in single extrapolation mode or in statistics mode).

In statistical mode, progress bar displays the estimated remaining duration. This duration is estimated by multiplying remaining extrapolations number by averaged past extrapolation durations. Averaged extrapolation duration is equal to the difference between current time and initial time, then divided by the number of performed extrapolations.


### 3.3. Open a LEO simulation example

The user can learn how to use STELA software with the help of a simulation example. A configurated file is available in the directory "\{installation directory\}/examples". In order to select the example, the user must use the STELA menu "File -> Open a new simulation...", and then select the example file "example_LEO_sim.xml".


Only files with the extension "*_sim.xml" can be opened by STELA.
The rest of the current chapter will consider this simulation example in order to describe the different GUI views.

### 3.4. Parameters of a LEO simulation

### 3.4.1. Navigation

The left part of the STELA window allows the user to navigate and to select the STELA window.


### 3.4.2. General Parameters

The following image displays a view of the General Parameters window. These parameters are listed below.
Note that tooltips are available for the simulation parameters. They appear as soon as the mouse is pointing the name of a parameter.
Warning: special characters (such as ) should not be added to text field, as STELA will not be able to save/load them.


The user may fill in:

1. the simulation mode:

- the default mode performs a single extrapolation
- the iterative mode performs an iterative search of an initial orbit with a given orbit lifetime (see §Iterative mode for LEO and GEO orbits)

The simulation information:
2. the author name
3. comments
4. the simulation duration in years. A year lasts 365.25 days ( 365 days and 6 hours).

## The spacecraft main characteristics:

5. its name
6. its total mass (kg)
7. its mean cross sectional reflecting area $\left(\mathrm{m}^{2}\right)$
8. its reflectivity coefficient
9. its mean cross sectional drag area ( $\mathrm{m}^{2}$ )
10. its drag coefficient Cd which may be defined:

- by an input file ("stela_drag_coefficient", see Appendix A.1.)

The atmospheric drag settings:
12. the atmospheric model (NRLMSISE-00, US76 or Jacchia 77) to be used for atmospheric density computation
13. the solar activity : it is an entry for the NRLMSISE-00 and Jacchia 77 atmospheric models. The solar activity can be defined:

- with a solar activity file ("stela_solar_activity", report to Appendix A.2.) that contains daily information made of the daily solar flux (sfu), the mean solar flux (sfu), and the geomagnetic 3-Hour index Ap (eight values defined for 24 hours)
- with a solar activity file from DAS ("solarflux_table.txt", report to Appendix A.2.) that contains daily information made of the daily solar flux (sfu). The Ap index are not defined in this file and then are set by default to 9 (value tunable in the stela_advanced_parameters file). - with a "mean constant" normalized solar activity computed from the ballistic coefficient of the spacecraft and the apoapsis altitude of the initial orbit (see Appendix A.6. for more information)
- with user defined constant values:

14. solar flux (sfu)
15. geomagnetic index Ap

## The initial state:

16. the nature of the initial orbital parameters (mean or osculating see §Orbital elements)
17. the type of the initial orbital parameters (see §Orbital elements)
18. the frame in which the initial orbit is expressed (see §Frames)

## The orbit parameters:

19. the calendar date of the initial orbit (see §Time scales)

20 to 25 . the six parameters describing the orbit.
Be careful, parameters are automatically rounded to 12 digits and angles are restricted to interval [ $0^{\circ} ; 360^{\circ}$ [ when entered by user.
26. The output ephemeris step that will be used for plots and output ephemeris file saving. Be careful, if you record an osculating ephemeris with a small timestep (ex: 1h) with a force model including the SRP, the saving time will be long (about 1 min for a 100 year propagation with a 1 h ephemeris timestep).

For Terrestrial Frozen at Epoch frame, two additional fields are displayed:


Fields "Freeze epoch" and "Reference longitude" are two parameters of Terrestrial Frozen at Epoch frame (see §Frames).

When the software runs in iterative mode (1), the following parameters shall be defined:

- The type of iteration mode : eccentric orbit (tunes the periapsis altitude) or frozen orbit (tunes the semi-major axis and the frozen eccentricity value), see §Iterative mode for LEO and GEO orbits (2)
- The expected lifetime of the searched orbit (3)



### 3.4.3. Advanced Parameters

The advanced default parameters contain recommended values.


The "Advanced Parameters" view contains :

1. the integration step
2. a flag used to enable/disable the atmospheric drag force
3. the number of points for the Simpson quadrature (used for the modeling of the atmospheric drag force, see §Algorithm features)
4. the number of integration steps where the atmospheric drag force is considered to be constant (therefore, the drag force recomputation occurs every N integration steps)
5. a flag used to enable/disable the Solar Radiation Pressure (SRP) perturbation
6. the number of points for the Simpson quadrature (used for the modeling of SRP, see §Algorithm features)
7. a flag used to enable/disable the Sun perturbations
8. a flag used to enable/disable the Moon perturbations
9. a flag used to enable/disable the Earth potential zonal perturbation
10. the zonal harmonics degree of Earth gravity model
11. a flag used to enable/disable the Earth potential tesseral perturbation
12. the tesseral harmonics order of Earth gravity model
13. the minimum period used in the tesseral effect computation. The tesseral effect is taken into account if its effect has a period greater than the given value, expressed as a multiple of the integration step.
14. a flag used to enable/disable the solid tides perturbation (combining the effects of Sun and Moon).
15. the reentry altitude. The spacecraft enters the atmosphere when the perigee altitude of its orbit goes bellow this value.
16. the delay TT-UT1 (used in frame transformations, see § Time scales, and when importing TLE).

The right part of the view appears only if the software runs in iterative mode. Then, the right part is divided into :
17. the definition of the algorithm convergence threshold.
18. the maximum delta between the expected lifetime entered by the user and the extrapolation duration computed by the propagator.

### 3.5. Open a GEO simulation example

The user can learn how to use STELA software with the help of a simulation example. A configurated file is available in the directory "\{installation directory\}/examples". In order to select the example, the user must use the STELA menu "File -> Open a new simulation...", and then select the example file "example_GEO_sim.xml".


Only files with the extension "*_sim.xml" can be opened by STELA.
The rest of the current chapter will consider this simulation example in order to describe the different GUI views.

### 3.6. Parameters of a GEO simulation

### 3.6.1. Navigation

The left part of the STELA window allows the user to navigate and to select the STELA window.


### 3.6.2. General Parameters

The following image displays a view of the General Parameters window. These parameters are listed below.
Note that tooltips are available for the simulation parameters. They appear as soon as the mouse is pointing the name of a parameter.
Warning: special characters (such as ) should not be added to text field, as STELA will not be able to save/load it.


The user may fill in:

1. the simulation mode:

- the default mode performs a single extrapolation
- the iterative mode performs an iterative search of an initial orbit that will stay above a minimal altitude during a given exclusion time (see §Iterative mode for LEO and GEO orbits )

The simulation information:
2. the author name
3. comments
4. the simulation duration in years. A year lasts 365.25 days ( 365 days and 6 hours).

The spacecraft main characteristics:
5. its name
6. its total mass (kg)
7. its mean cross sectional reflecting area ( $\mathrm{m}^{2}$ )
8. its reflectivity coefficient
9. its mean cross sectional drag area $\left(\mathrm{m}^{2}\right)$
10. its drag coefficient Cd which may be defined:

- by an input file ("stela_drag_coefficient", see Appendix A.1.)

11. as a constant value given in field

- from Cook formula

The atmospheric drag settings:
Note that atmospheric density becomes very low for altitudes above 2500km and therefore atmospheric drag can be neglected. Nevertheless, a GEO user still has the possibility to activate it.
12. the atmospheric model (NRLMSISE-00, US76 or Jacchia 77) to be used for atmospheric density computation
13. the solar activity : it is an entry for the NRLMSISE-00 and Jacchia 77 atmospheric models. The solar activity can be defined:

- with a solar activity file ("stela_solar_activity", report to Appendix A.2.) that contains daily information made of the daily solar flux (sfu), the mean solar flux (sfu), and the geomagnetic 3-Hour index Ap (eight values defined for 24 hours)
- with a solar activity file from DAS ("solarflux_table.txt", report to Appendix A.2.) that contains daily information made of the daily solar flux (sfu). The AP coefficients are not defined in this file and then are set by default to 9 .
- with user defined constant values:

14. solar flux (sfu)
15. geomagnetic index Ap

## The initial state:

16. the nature of the initial orbital parameters (mean or osculating, see §Orbital elements)

Be careful, when SRP is active, nature conversion relies on S/M ratio and reflectivity coefficient, and when Sun or Moon perturbation is active it relies on the initial date.
17. the type of the initial orbital parameters (see §Orbital elements)
18. the frame in which the initial orbit is expressed (see §Frames)

## The orbit parameters:

19. the calendar date of the initial orbit (see §Time scales)

20 to 25 . the six parameters describing the orbit.
Be careful, parameters are automatically rounded to 12 digits and angles are restricted to interval [ $0^{\circ} ; 360^{\circ}$ [ when entered by user.
26. The output ephemeris step that will be used for plots and output ephemeris file saving. Be careful, if you record an osculating ephemeris with a small timestep (ex: 1h) with a force model including the SRP, the saving time will be long (about 1 min for a 100 year propagation with a 1 h ephemeris timestep).

When the software runs in iterative mode (1), the following parameters shall be defined:

- The GEO region exclusion duration (2)
- The targetted eccentricity ex and ey (see §Iterative mode for LEO and GEO orbits)(3 and 4)
- The minimal perigee altitude above the GEO altitude that must not be reached during the exclusion duration (5)



### 3.6.3. Advanced Parameters

The advanced default parameters contain recommended values.


The "Advanced Parameters" view contains :

1. the integration step
2. a flag used to enable/disable the atmospheric drag force
3. the number of points for the Simpson quadrature (used for the modeling of the atmospheric drag force, see §Algorithm features)
4. the number of integration steps where the atmospheric drag force is considered to be constant (therefore, the drag force recomputation occurs every N integration steps)
5. a flag used to enable/disable the Solar Radiation Pressure (SRP) perturbation
6. the number of points for the Simpson quadrature (used for the modeling of SRP, see §Algorithm features)
7. a flag used to enable/disable the Sun perturbations
8. a flag used to enable/disable the Moon perturbations
9. a flag used to enable/disable the Earth potential zonal perturbation
10. the zonal harmonics degree of Earth gravity model
11. a flag used to enable/disable the Earth potential tesseral perturbation
12. the tesseral harmonics order of Earth gravity model
13. the minimum period used in the tesseral effect computation. The tesseral effect is taken into account if its effect has a period greater than the given value, expressed as a multiple of the integration step.
14. a flag used to enable/disable the Earth potential tesseral perturbation
15. a flag used to enable/disable the solid tides perturbation (combining the effects of Sun and Moon)
16. the reentry altitude. The spacecraft enters the atmosphere when the perigee altitude of its orbit goes bellow this value.
17. the delay TT-UT1(used in frame transformations, see § Time scales, and when importing TLE).

The right part of the view appears only if the software runs in iterative mode. Then, the right part contains :
17. the definition of the algorithm convergence threshold.

### 3.7. Open a GTO simulation example

The user can learn how to use STELA software with the help of a simulation example. A configurated file is available in the directory "\{installation directory\}/examples". In order to select the example, the user must use the STELA menu "File -> Open a new simulation...", and then select the example file "example_GTO_sim.xml".


Only files with the extension "*_sim.xml" can be opened by STELA.
The rest of the current chapter will consider this simulation example in order to describe the different GUI views.

### 3.8. Parameters of a GTO simulation

### 3.8.1. Navigation

The left part of the STELA window allows the user to navigate and to select the STELA window.


### 3.8.2. General Parameters

The following image displays a view of the General Parameters window. These parameters are listed below.
Note that tooltips are available for the simulation parameters. They appear as soon as the mouse is pointing the name of a parameter.
Warning: special characters (such as ) should not be added to text field, as STELA will not be able to save/load it.


The user may fill in:
The simulation information:

1. the author name
2. comments
3. the simulation duration in years. A year lasts 365.25 days ( 365 days and 6 hours).

## The spacecraft main characteristics:

4. its name
5. its total mass (kg)
6. its mean cross sectional reflecting area ( $\mathrm{m}^{2}$ )
7. its reflectivity coefficient
8. its mean cross sectional drag area ( $\mathrm{m}^{2}$ )
9. its drag coefficient Cd which may be defined:

- by an input file ("stela_drag_coefficient", see Appendix A.1.)

10. as a constant value given in field

- from Cook formula

The atmospheric drag settings:
11. the atmospheric model (NRLMSISE-00, US76 or Jacchia 77) to be used for atmospheric density computation
12. the solar activity : it is an entry for the NRLMSISE-00 and Jacchia 77 atmospheric models. The solar activity can be defined:

- with a solar activity file ("stela_solar_activity", report to Appendix A.2.) that contains daily information made of the daily solar flux (sfu), the mean solar flux (sfu), and the geomagnetic 3-Hour index Ap (eight values defined for 24 hours) - with a solar activity file from DAS ("solarflux_table.txt", report to Appendix A.2.) that contains daily information made of the daily solar flux (sfu). The Ap index are not defined in this file and then are set by default to 9 (value tunable in the stela_advanced_parameters file). - with user defined constant values:

13. solar flux (sfu)
14. geomagnetic index Ap
15. the statistics mode switch
16. the maximum number of runs in statistics mode

## The initial state:

17. the nature of the initial orbital parameters (mean or osculating see §Orbital elements)
18. the type of the initial orbital parameters (see §Orbital elements)
19. the frame in which the initial orbit is expressed (see §Frames)

## The orbit parameters:

20. the calendar date of the initial orbit (see §Time scales)

21 to 26 . the six parameters describing the orbit.
Be careful, parameters are automatically rounded to 12 digits and angles are restricted to interval [ $0^{\circ} ; 360^{\circ}$ [ when entered by user.
27. The output ephemeris step that will be used for plots and output ephemeris file saving. Note that, in GTO case, when the transition matrix computation is activated, this is also the step of the transition matrix ephemeris file. Be careful, if you record an osculating ephemeris with a small timestep (ex: 1h) with a force model including the SRP, the saving time will be long (about 1min for a 100 year propagation with a 1 h ephemeris timestep).

For Terrestrial Frozen at Epoch frame, two additional fields are displayed:


Fields "Freeze epoch" and "Reference longitude" are two parameters of Terrestrial Frozen at Epoch frame (see §Frames).

### 3.8.3. Advanced Parameters

The advanced default parameters contain recommended values.


The "Advanced Parameters" view contains :

1. the integration step
2. a flag used to enable/disable the atmospheric drag force
3. the number of points for the Simpson quadrature (used for the modeling of the atmospheric drag force, see §Algorithm features)
4. the number of integration steps where the atmospheric drag force is considered to be constant (therefore, the drag force recomputation occurs every N integration steps)
5. a flag used to enable/disable the Solar Radiation Pressure (SRP) perturbation
6. the number of points for the Simpson quadrature (used for the modeling of SRP, see §Algorithm features)
7. a flag used to enable/disable the Sun perturbations
8. a flag used to enable/disable the Moon perturbations
9. a flag used to enable/disable the Earth potential zonal perturbation
10. the zonal harmonics degree of Earth gravity model
11. a flag used to enable/disable the Earth potential tesseral perturbation
12. the tesseral harmonics order of Earth gravity model
13. the minimum period used in the tesseral effect computation. The tesseral effect is taken into account if its effect has a period greater than the given value, expressed as a multiple of the integration step.
14. a flag used to enable/disable the solid tides perturbation (combining the effects of Sun and Moon)
15. the reentry altitude. The spacecraft enters the atmosphere when the perigee altitude of its orbit goes bellow this value.
16. the delay TT-UT1 (used in frame transformations, see § Time scales, and when importing TLE).

### 3.8.4. Statistics Parameters

The statistics view contains all parameters related to the statistical mode.


The "Statistics" view contains :

1. the stop mode: automatic or manual (see 4.1. Termination Criteria for more information)
2. the initial seed. It can be regenerated using the button "Generate" on the left
3. the date dispersion panel
4. the hour dispersion panel
5. the mass dispersion panel
6. the solar activity dispersion panel (if atmospheric model is NRLMSISE-00 or Jacchia 77)
7. the orbit dispersion panel
8. the orbital parameters dispersion matrix (correlation, covariance or no dispersion). In the case of correlation, a vector of standard deviation/delta has to be provided as well.
9. the orbital parameters dispersion type
10. the nature of the orbital parameters dispersion
11. the type of the orbital parameters dispersion. Initial bulletin will be converted into it before dispersion
12. the frame in which the initial bulletin is expressed in. This is simply a reminder and cannot be changed
13. the covariance/correlation matrix
14. the multiprocessing mode : activate/deactivate parallel computing
15. the number of processes that will be launched. For an optimal execution, number of processes should be equal to the number of computer cores. Be careful, in some cases, your processor can be run in hyperthreading mode. In this case, divide the number by two. See also your processor documentation.
16. the reflecting area dispersion panel
17. the reflectivity coefficient dispersion panel
18. the drag area dispersion panel
19. the drag coefficient dispersion panel

For more information on available dispersions, see 5.9. "Dispersions used for statistical analysis".
NB: Mean/Nominal values are those from the General Panel.

### 3.9. Results of a simulation

### 3.9.1. Summary for single extrapolation mode

When a simulation ends STELA software automatically switch to the Results/Summary view that is divided in two topics:

- The left part describes the final orbit state as followed :
- the nature, type and frame are reminded to the user
- the final orbit parameters (date, position and velocity)
- The right part reports the compliance with criteria through the plot of the effective simulation duration and the status of the four criteria (see § Assessing Compliance with LEO, GEO \& GTO Protected Region criteria). If a criterion is violated the first violation date or the estimated lifetime is indicated.

The user shall keep in mind that the Criteria are evaluated through the osculating parameters expressed at evaluation points along the orbit, see § Assessing Compliance with LEO \& GEO Protected Region criteria, whereas the orbital parameters given in the left part of the view come from the last point computed by the integrator which is not necessary at the perigee or the apogee and may be given through mean parameters. It explains that the crossing of a protected region or the reentry of the spacecraft may not be blindingly obvious by looking at the final orbit parameters.

If C2 criterion is compliant, the minimum distance to LEO protected region is displayed.
If C3 criterion is compliant: the minimum distance to GEO altitude and the corresponding latitude are displayed, as well as the last date in the GEO protected region (for orbits with a GEO standard inclination; threshold value is given in § Physical and key parameters).
If not compliant: the date of the first criterion violation is displayed as well as the last date in the GEO protected region if before a time limit $\mathrm{t}_{\text {max }}$ (a few years; value is given in § Physical and key parameters ).

If C4 criterion is compliant, the minimum distance to GEO altitude and the corresponding latitude are displayed (for orbits with GEO standard inclination; threshold value is given in § Physical and key parameters).



## Warning:

- See next paragraph for GTO orbits.
- For LEO orbits with specific inclination, the following message may appear : "Due to resonance phenomena, extrapolation results may be very sensitive to initial parameters (See User Manual)"; See paragraph Control of validity domain for more details.


### 3.9.2. Limitations of the GTO single extrapolation mode

For GTO orbits, the extrapolation results may be very sensitive to the initial conditions or to the parameters of models. A tiny modification of the initial conditions or the computation parameters ( $\mathrm{S} / \mathrm{m}$ ratio, drag and SRP coefficients, solar activity ...) might end up with significant different results. The following plot shows an example of the evolution of the semi-major axis of a GTO orbit and the difference in reentry dates for the same initial orbit, only slightly changing the $\mathrm{S} / \mathrm{m}$ ratio:


It is clear that a modification of less than one percent of one parameter of the initial configuration can change the reentry date by more than 10 years.
This sensitive behaviour is due to the sun-moon perturbation and to resonance phenomena. In order to get a reliable status regarding the criteria validation, one does not simply extrapolate STELA once. A statistical computation using the "statistical mode" (through GUI or in batch mode) is to be done in order to obtain relevant results. This is the reason why a warning message and only an orange "Not Reliable" status appear when using the GTO single extrapolation mode. See $\S$ Ref 6 and §Ref 8 for more information on these resonance phenomena.

### 3.9.3. Summary for iterative mode in LEO

When a simulation in iterative mode ends STELA software automatically switches to the Results/Summary view that is divided in three topics :

- The top part displays the effective lifetime of the adjusted initial orbit. This lifetime is equal or smaller (with respect to the algorithm convergence threshold) than the expected lifetime given by the user. The number of iterations needed to adjust the initial state is also indicated.
- The bottom part describes the adjusted initial orbit state as followed :
- the nature, type and frame are reminded to the user
- the orbit parameters (date, position and velocity)

A button is provided to copy the adjusted initial state to general parameters view in order to perform a single extrapolation to check the compliance with protected region criteria.


### 3.9.4. Summary for iterative mode in GEO

When a simulation in iterative mode ends STELA software automatically switches to the Results/Summary view that is divided in three topics :

- The top part displays the minimal altitude minus the GEO radius (see § "Protected Region" criteria for computation method of C 4 criterion) reached by the propagated adjusted initial orbit. This altitude is equal or bigger (with respect to the algorithm convergence threshold) than the minimal altitude given by the user. The number of iterations needed to adjust the initial state is also indicated.
- The bottom part describes the adjusted initial orbit state as followed :
- the nature, type and frame are reminded to the user
- the orbit parameters (date, position and velocity)

A button is provided to copy the adjusted initial state to general parameters view in order to perform a single extrapolation to check the compliance with protected region criteria.

## Iteration report summary

Final min. perigee altitude / GEO

## Frame: CIRF

```
Orbit parameters
    Mate 
```

Copy adjusted initial state
to general parameters view

### 3.9.5. Summary for GTO statistics mode

When a statistics simulation starts STELA software automatically switches to the Results/Summary view that is divided in two topics :

- The left part display the four graphs :
- The $95 \%$ confidence interval and observed probability vs execution number for SC1 or SC2 criterion (depending on applicable criterion)
- The lifetime cumulative distribution function for SC 1 criterion
- The $95 \%$ confidence interval and observed probability vs execution number for SC3 criterion
- The $95 \%$ confidence interval and observed probability vs execution number for SC 4 criterion
- The right part reports the compliance with the four statistical criteria (see § Assessing Compliance with LEO, GEO \& GTO Protected Region criteria).

For each criterion, the value of the lower bound (if compliant) / upper bound (if not compliant) of Wilson confidence interval is displayed. This is the probability value used to check the compliance.


### 3.10. Output data and Plots

### 3.10.1. Output files

When the user saves a simulation three output files are generated (four when using GTO statistical mode)

- the simulation file, with the extension *_sim.xml. It is an xml file that contains all the simulation information and can be re-loaded by STELA software.
- the report file, with the extension *_sim.txt. It is a copy of the simulation file with round results as GUI but in a text format. In particular, if the simulation has already been run the report file contains the summary of the compliance with the protected LEO \& GEO region criteria. The report file can not be re-loaded by STELA. A display of a report file is available in Appendix A.4.
- the $\log$ file, with the extension *_log.txt. It is a text file that contains all the $\log$ messages that have been displayed. The log file can not be re-loaded by STELA
- GTO statistical mode only: the statistical text file, with the extension *_sim_stat.txt. It contains a matrix of all simulations generated with the statistical mode (inputs and computed outputs)
- GTO mode only: the state transition matrix file, with the extension *_stm.txt, when this computation is enabled by the user. This is an ephemeris file containing the orbital elements and the state transition matrix. See Appendix A.7.


### 3.10.2. Single extrapolation mode output view

The Output view is intended, once the simulation is properly done, to allow the user to plot some orbit parameters or to save an ephemeris file.


### 3.10.2.1 Plots

The user has the possibility to choose between eleven plots of results :

- the values of Perigee and Apogee altitude (km) vs the simulation duration (duration is relative to the start time t 0 and expressed in years)
- the values of Perigee altitude (km) vs the simulation duration (duration is relative to the start time t0 and expressed in years)
- the values of Apogee altitude (km) vs the simulation duration (duration is relative to the start time t0 and expressed in years)
- the values of Semi-major axis (km) vs the simulation duration (duration is relative to the start time t0 and expressed in years)
- the values of Eccentricity vs the simulation duration (duration is relative to the start time t 0 and expressed in years)
- the eccentricity vector ey vs ex for quasi-circular orbit, with ex=e. $\cos (\omega)$ and ey=e. $\sin (\omega)$, that makes perfect sense for LEO orbit
- the eccentricity vector ey vs ex for quasi-circular and quasi-equatorial orbits, with ex=e. $\cos (\omega+\Omega)$ and ey=e. $\sin (\omega+\Omega)$, that makes perfect sense for GEO orbit
- the values of Inclination (deg) vs the simulation duration (duration is relative to the start time t 0 and expressed in years)
- the inclination vector iy vs ix for quasi-equatorial orbits, with $\mathrm{ix}=\sin (\mathrm{i} / 2) \cdot \cos (\Omega)$ and $\mathrm{iy}=\sin (\mathrm{i} / 2) \cdot \sin ($ $\Omega$ ), that makes perfect sense for GEO orbits
- the values of Argument of Perigee (deg) vs the simulation duration (duration is relative to the start time t0 and expressed in years)
- the values of Right ascension of the ascending node (deg) vs the simulation duration (duration is relative to the start time $t 0$ and expressed in years)
- the values of the sum $++\mathrm{M}(\mathrm{deg})$ vs the simulation duration (duration is relative to the start time t0 and expressed in years)
- The solar flux F10.7 used in the extrapolation for atmospheric density computation
- The geomagnetic index Ap used in the extrapolation for atmospheric density computation

Please note that :

- the plotted orbit parameters are mean parameters
- the plotted orbit parameters are given in the STELA integration frame that is the CIRF frame (see §Frames)
- the plotted perigee and apogee altitude are computed for each ephemeris point as followed :
- $\mathrm{ha}=\mathrm{a} .(1+\mathrm{e})-6,378 \mathrm{~km}$
- $\mathrm{hp}=\mathrm{a} .(1-\mathrm{e})-6,378 \mathrm{~km}$
- the solar flux and geomagnetic index can be output only if either NRLMSISE-00 or Jacchia atmospheric models has been used during the extrapolation.

Keeping in mind that the Criteria are evaluated through the osculating parameters, it explains that the crossing of a protected region or the reentry of the spacecraft may not be blindingly obvious by looking at the plots.

It is easy to change the axes or title names or to zoom in and out by making a right click on the new window.

### 3.10.2.2 Ephemeris file

The user can save the computed ephemeris points in a text file with the extension *_eph.txt. The ephemeris file cannot be re-loaded by STELA.

The ephemeris can be saved in two different formats:

- a CCSDS-compliant format called CCDDS OEM (see "ORBIT DATA MESSAGES", CCSDS 502.0-B-2);
- a STELA format called STELA OEM which uses Modified Julian Days (see 5.3.8.).

The user can also choose:

- The nature of the parameters (Mean or Osculating),
- The type of the parameters (Cartesian, Keplerian or Equinoctial),
- The frame in which the parameters are expressed. For the Terrestrial Frozen at Epoch frame, two additional fields are displayed (same behavior as to choose the initial state).

A display of both formats of ephemeris files is available in Appendix A.3.

### 3.10.3. Statistical mode output view

### 3.10.3.1 Plots and complementary results

This section displays general results about the simulation, and complementary results about each criterion.


The user has the possibility to choose between six plots of results :

- Lifetime vs execution number
- Lifetime distribution histogram
- Lifetime cumulative distribution function
- The $95 \%$ confidence interval and observed probability vs execution number for SC1 or SC2 (depending on applicable criterion)
- The $95 \%$ confidence interval and observed probability vs execution number for SC3
- The $95 \%$ confidence interval and observed probability vs execution number for SC4

It is easy to change the axes or title names or to zoom in and out by making a right click on the new window.

### 3.10.3.2 Ephemeris output

When running a statistical analysis, the user has the possibility to output the ephemeris files and xml context files of every run by modifying the associated parameter in the advanced parameters file. Output path, nature, type and frame of the parameters can also be chosen through this file.

### 3.11. Tools

### 3.11.1. Compute Mean Area

The user can select "Tools" in order to start the STELA tool "Mean Area Computation"


The Mean Area Computation Tool allows the user to draw a simplify model of the Spacecraft and to compute the effective cross sectional area to be used in drag force or solar radiation pressure computation. The cross sectional area unit is the square of the input unit.


The spacecraft is modelled as a collection of shapes, chosen from the following list:

- Sphere
- Rectangle
- Cuboid
- Triangle
- Triangle (coord)
- Truncated cone
- Hollowed truncated cone (warning: thickness smaller than radius may lead to wrong results)

When created, each shape (except from the triangle which can be given through the coordinates of its vertices) is initially defined by (see screenshot):

- one reference point (red cross) set to $(0,0,0)$
- one shape-bound orientation vector (green vector) set to the z vector $(0,0,1)$
- its dimensions (blue segments) set to 1

To customize each shape, the user may:

- translate the shape by repositionning the reference point ("position vector" field)
- rotate the shape by changing (no need to keep the vector unitary) the coordinates of the shape-bound orientation vector ("orientation vector" field): the rotation which transforms the default orientation vector $(\vec{z})$ into the user-defined one $(\vec{v})$ is applied to the whole shape (the position of the orientation vector remains unchanged with respect to the shape). The characteristics of this rotation are:
- axis $=\vec{z} \wedge \vec{v} /\|\vec{v}\|_{\text {(if }} \vec{z} \wedge \vec{v}=\overrightarrow{0}$, an arbitrary axis is chosen and if $\|\vec{v}\|=0$, no rotation is applied )
- angle $=\operatorname{arcos}(\vec{z} \cdot \vec{v} /\|\vec{v}\|)$
- rotate the shape on the orientation vector by a given angle ("angle" field)
- resize the shape by filling in the dimensions fields with its actual dimensions

For the Triangle (coord) object:
This shape is defined by the 3 dimensions coordinates of its vertices. The reference point and the orientation vector will be automatically deduced from this definition.

The current shape is outlined in red in a 2-D view showing the three orthogonal view (Y-X, Y-Z, X-Z).
It is also available in a second tab, showing a 3-D view (see screenshot). The main functionalities of this view are:

- Zoom-in: Mouse wheel up
- Zoom-out: Mouse wheel down
- Rotation: Left click and hold
- Translation: Right click and hold


Beneath the 3D-view is a button (autoscale) that allows the user to set the zoom to fit the object.
Autoscale is also automatically performed when adding or removing an object.
The computation of the Mean area can be done considering several orientation model:

- Random tumbling: the spacecraft attitude is variable and has no particular direction
- Spin: the spacecraft has a spin movement along one particular axis (case of gravity gradient for example). The user can define:
- the rotation axis vector
- the direction of observation
- Fixed orientation: the user simply defines the direction of observation, the mean area will be the cross sectional area perpendicular to this direction

The computation of the Mean Area is done as follows:

- A projection area perpendicular to an observation direction is designed so that this area contains the projection of the object whatever its orientation is.
- The area is cut into $n p$ pixels. $n p$, the number of pixels of projection area, can be modified by the user through the Advanced Parameter tab. It is recommended to use a high number of pixels in order to have a reliable computed mean area.
- Then a ray is shooted perpendicularly from each pixel of the scene. The pixel is switched on or off whether the ray intersects or not one of the shapes
- The fraction of switched-on pixels multiplied by the area of the scene is the cross sectional area from the current direction of observation
- The direction of observation is then changed to take into account the orientation model given by the user. The number of directions can be defined by the user through the Advanced Parameter tab.


SCENE

The model can be saved in a XML file with the extension *_shap.xml. This file can be re-opened by the STELA Mean Area Computation tool.

A text file with extension *_shap.txt is saved simultaneously. This file contains the matrix of computed area following the pattern:
\{ azimuth elevation area \}
where:

- azimuth: angle between X and the direction of observation projected in the $(\mathrm{X}, \mathrm{Y})$ plane
- elevation: angle between the direction of observation and the (X,Y) plane
- area: cross sectional area seen from the direction of observation


### 3.11.2. Two Line Elements tool

STELA provides users with a Two-Line Elements (hereafter called TLE) conversion tool, based on the SGP4/SDP4 theory.


- The user can convert one or several TLE at once into a format compatible with STELA.
- Conversion can be performed in osculating parameters or in mean parameters according to SGP4/SDP4 theory.
- Once converted, the user can select an output state and retrieve it in the initial TLE list.
- The selected state can also be copied in current STELA session, assuming that a simulation has previously been created. If so, the bulletin is copied in the orbital parameters frame, the International Designator in the space object's name text area and the ballistic coefficient in the commentary text area.



One must pay attention to the fact that converted state's date is defined in the UTC Time System. Indeed, STELA default Time System is UT1; therefore while copying the selected state into current STELA session, a Time System conversion (UTC to UT1) is performed to meet the STELA standards, using the "TT minus UT1" value of the Advanced Parameters panel.

Two options are provided for the conversion from TLE data:

- Conversion from TLE to osculating orbit parameters.

This conversion is done using SGP4-SDP4 theory as described in §Ref 10. This conversion is not valid for high eccentricities due to limitations in the SGP4-SDP4 short period model. It is recommended for quasi circular orbits when importing into STELA session.

- Conversion to SGP4-SDP4 mean parameters, Brouwer convention.

This conversion is valid for all eccentricities since no short periods are added to the orbital elements. It is recommended for high eccentricity orbits when importing into STELA session. It includes a conversion of the Kozai mean motion of TLE orbital products to the Brouwer mean motion (see TLE conversion §5.13).

## 4. Assessing Compliance with LEO \& GEO Protected Region criteria

When the impossibility to carry out a controlled reentry is duly proven, the conformity with the French Space Operations Act requirements is evaluated through four "Protected LEO \& GEO Regions" criteria. Violation of these criteria never arouses the end of simulation. A simulation will stop only when one of the two "termination criteria" is reached.

The protected LEO \& GEO Regions are defined as follows :

- Protected LEO region extends from the Earth surface up to the geocentric altitude of 2,000 km.
- Protected GEO region is defined by boundaries in latitude ( $\mathbf{- 1 5} \mathbf{~ d e g ~ ; ~ + ~} \mathbf{1 5} \mathbf{~ d e g}$ ) and orbit radius $( \pm 200 \mathrm{~km}$ with respect to the geocentric altitude of $35,786 \mathrm{~km}$ ).



### 4.1. Termination criteria

Two termination criteria can trigger the end of simulation for a single extrapolation. Three termination criteria are specific to the statistical mode.

### 4.1.1. Termination criterion TC1

## Title

[^0]The simulation stops when the simulation duration is reached. The simulation duration is an input parameter defined in the GUI.

### 4.1.2. Termination criterion TC2

## Title

"Termination criterion TC2 : the space object has begun its atmospheric reentry"

## Method of use

The simulation stops as soon as the spacecraft enters the reentry atmosphere. The reentry altitude is defined by the user with the help of GUI (default value is 120 km for LEO orbits and 80 km for GTO orbits). The termination criterion TC2 triggers when the periapsis altitude becomes lower than the reentry altitude. The periapsis altitude is computed by the software at every integration step as follows :

- The mean parameters are propagated till the periapsis (mean anomaly set to zero)
- Mean parameters at the periapsis are converted to Osculating parameters
- The geocentric periapsis altitude is : hp = a_osc.( $1-\mathrm{e} \_$osc) $-6,378 \mathrm{~km}$


### 4.1.3. Statistical Termination criterion STC1 (Statistical mode)

## Title

"Statistical Termination criterion STC1 : the maximum number of single extrapolations given by the user has been reached".

## Method of use

The statistical analysis stops when the maximum number of single extrapolations is reached. The maximum number of single extrapolations is an input parameter defined in the GUI.

### 4.1.4. Statistical Termination criterion STC2 (Statistical mode)

## Title

"Statistical Termination criterion STC2 : Automatic stop".

## Method of use

When the automatic stop mode is chosen in the GUI, the Statistical Analysis stops when the number of single extrapolations is great enough so that all the Statistical Criteria are defined (status different from "not computable"): if the extrapolation duration is lower than 100 years, only SC1 needs to be defined to trigger STC2. Otherwise, all four statistical criteria need to have a defined status.

### 4.1.5. Statistical Termination criterion STC3 (Statistical mode)

## Title

"Statistical Termination criterion STC3 : Statistical Analysis stopped by the user".

## Method of use

When clicking the Stop button in the progression Window, Statistical Analysis will stop as soon as all the extrapolations still being computed are finished.

## 4.2. 'Protected Region'" criteria

### 4.2.1. C1 criterion : 'Lifetime < 25 years"

## Title

"The C 1 criterion is violated if the spacecraft lifetime (end of simulation date - beginning date) exceeds 25 years."

## Method of use

It means that the C 1 criterion is violated if the spacecraft object begins its reentry more than 25 years after the initial date. The C1 criterion is computable only at the end of simulation and is combined with the termination criterion TC2 (reentry in the atmosphere) :

- if TC2 has been reached and the effective simulation duration is less than 25 years C 1 is fulfilled
- if TC2 has been reached and the effective simulation duration is more than 25 years C1 is violated
- if TC2 has not been reached and the simulation duration is more than 25 years C 1 is violated
- if TC2 has not been reached and the simulation duration is less than 25 years C1 is not computable


### 4.2.2. C2 criterion : "No LEO crossing within 100 years"

## Title

"The C 2 criterion is violated if the geocentric periapsis altitude reaches an altitude lower than $2,000+\mathrm{h}$ km during the first 100 extrapolation years."
$h$ is a margin to be considered due to the fact that the modelization is a simplified modelization wrt precise reference numerical propagators taking into account a full dynamical model. Its values are given in § Physical and key parameters.

## Method of use

The C2 criterion is checked at every integration step as follows :

- The mean parameters are propagated up to the periapsis (mean anomaly set to zero)
- Mean parameters at the periapsis are converted to osculating parameters
- The geocentric periapsis altitude is : $\mathrm{hp}=\mathrm{a}$ _osc. $\left(1-\mathrm{e} \_\right.$osc) $)-6,378 \mathrm{~km}$
- If the geocentric altitude is higher than $(2,000+\mathrm{h}) \mathrm{km}$, the spacecraft is considered as far enough from the LEO protected region: the C 2 criterion is not violated.
- If the geocentric altitude is lower than $(2,000+\mathrm{h}) \mathrm{km}$, the C 2 criterion is violated.


### 4.2.3. C3 criterion : "No GEO crossing between 1 and 100 years"

## Title

"The C3 criterion is violated if the following conditions are fulfilled between the first and the hundredth (included) extrapolation year :

- (35,786 - (200+h) km) < altitude < (35,786 + (200+h) km)
- -15 deg < latitude < + 15 deg"
h is a margin to be considered due to the fact that the modelization is a simplified modelization, with respect to precise reference numerical propagators taking into account a full dynamical model. Its value is given in § Physical and key parameters.


## Method of use

The method is the same as for the C 4 criteria. See next paragraph.

### 4.2.4. C4 criterion : 'No GEO crossing within 100 years"

## Title

"The C 4 criterion is violated if the following conditions are fulfilled during the first hundred extrapolation years :

- $(35,786-(200+\mathrm{h}) \mathrm{km})<$ altitude $<(35,786+(200+\mathrm{h}) \mathrm{km})$
- -15 deg < latitude < $+15 \mathrm{deg} "$
h is a margin to be considered due to the fact that the modelization is a simplified modelization, with respect to precise reference numerical propagators taking into account a full dynamical model. Its value is given in § Physical and key parameters.


## Method of use

The mean parameters are first propagated up to the periapsis and the apoapsis (mean anomaly set to zero and pi). Then they are converted to osculating parameters. The geocentric periapsis and apoapsis altitudes are:

- $\mathrm{zp}=\mathrm{a}$ _osc.( 1 - e_osc) $-6,378 \mathrm{~km}$
- $\mathrm{za}=$ a_osc.( $1+\mathrm{e}$ _osc) $-6,378 \mathrm{~km}$.

If za < 35786-(200+h) or zp > 35786+(200+h):

- Then the criterion is not violated,
- Else if the mean inclination is below 15 deg or above 165 deg:
- Then the criteria is violated,
- Else the 4 mean anomalies corresponding to the $15^{\circ}$ latitude points are computed to compare the corresponding osculating altitudes to $35786-(200+\mathrm{h})$ and $35786+(200+\mathrm{h})$. Depending on these 4 osculating altitudes, the criteria may be violated or not.

The geocentric altitudes and latitudes of these osculating points are saved.

### 4.2.5. Statistical SC1 criterion : "Lifetime < 25 years with a probability of pSC 1 "

## Title

"The SC1 criterion is compliant if the C 1 criterion is compliant with a probability of pSC 1 "

## Method of use

SC1 is applicable when nominal (non dispersed) orbit crosses Region A. pSC 1 is equal to 0.9 . The computation method is explained in paragraph 4.2.9.
4.2.6. Statistical SC2 criterion : "No LEO crossing within 100 years with a probability of pSC 2 "

Title
"The SC2 criterion is compliant if the C 2 criterion is compliant with a probability of pSC 2 "

## Method of use

SC2 is applicable when nominal (non dispersed) orbit does not cross Region A.
pSC 2 is equal to 0.9. The computation method is explained in paragraph 4.2.9.
4.2.7. Statistical SC3 criterion : 'No GEO crossing between 1 and 100 years with a probability of pSC3"

## Title

"The SC3 criterion is compliant if the C3 criterion is compliant with a probability of pSC3."

## Method of use

pSC 3 is equal to 0.9. The computation method is explained in paragraph 4.2.9.
4.2.8. Statistical SC4 criterion : "No GEO crossing within 100 years with a probability of pSC 4 "

## Title

"The SC4 criterion is compliant if the C 4 criterion is compliant with a probability of pSC 4. ."

## Method of use

pSC 4 is equal to 0.9 . The computation method is explained in paragraph 4.2.9.

### 4.2.9. Method used to compute statistical criteria

All statistical criteria are computed following the same method. Let's consider the computation of SCi criterion:

During the Statistical Analysis, a number n of single extrapolations has been runned. Status of Ci criterion for each single run is either OK (if "Compliant") or NOK (otherwise).

The probability of SCi being compliant is based on the the number of OK runs over the total number of runs ( n ). This probability is then surrounded by the Wilson Confidence Interval with continuity correction of the Binomial law. The bounds of this interval are used to check the compliance. For more information about this topic, please see $\S$ Ref 9 .

The upper and lower limits of this interval are given by the following formula:

$$
\begin{aligned}
& p 1=\frac{2 n f+u_{\alpha / 2}{ }^{2}-1-u_{\alpha / 2} \sqrt{u_{\alpha / 2}{ }^{2}-2-1 / n+4 f(n(1-f)+1)}}{2\left(n+u_{\alpha / 2}{ }^{2}\right)} \\
& p 2=\frac{2 n f+u_{\alpha / 2}{ }^{2}+1+u_{\alpha / 2} \sqrt{u_{\alpha / 2}{ }^{2}+2-1 / n+4 f(n(1-f)-1)}}{2\left(n+u_{\alpha / 2}{ }^{2}\right)}
\end{aligned}
$$

With
. $u_{\alpha / 2}=\Phi^{-1}(1-\alpha / 2)=1.96$ for a confidence interval of $95 \%$. $\Phi$ : cumulative normal distribution function

- $\mathrm{f}=$ observed probability of SCi being compliant on the available distribution (based on n single extrapolations): number of OK runs divided by n .
- $\mathrm{n}=$ number of single extrapolations

In order to conclude on a criterion status, $n$ has to be greater than a minimum value $n m i n$ depending on the confidence rate (see $\S$ Ref 9 ).

$$
n_{\min }=\frac{1}{4} \frac{2 \cdot p S C i \cdot u_{\alpha / 2}^{2}+2+2 \sqrt{p S C i^{2} \cdot u_{\alpha / 2}{ }^{4}+2 \cdot p S C i \cdot u_{\alpha / 2}^{2}}}{1-p S C i}
$$

The following graph displays the result of the analysis for SCi criterion (SCi criterion needs to be verified with a pSCi probability):


The $95 \%$ confidence interval gives the interval which has a 0.95 probability to contain the converged probability of SCi being compliant.

As soon as the lower limit of the confidence interval (p1) is above the Statistical Criterion probability ( pSCi , for instance 0.9 for SC 1 ), SCi becomes "Compliant". Conversely, if the upper limit p 2 is below the pSCi threshold, the Statistical Criterion SCi is "Not Compliant". When neither p1 nor p2 has crossed pSCi , the status is not computable and needs more extrapolations to be able to conclude.

NB: In the very unlikely case where the limits p 1 and p 2 are both crossing pSCi , the criterion is said not computable as well, but increasing the number of single extrapolations will not be useful to solve the problem as the results displayed are likely to be among the $5 \%$ of cases out of the confidence interval. In this very case, there is no other choice but to replay the statistical analysis changing the initial seed.

The results displayed by Stela can be summarized in the following table:

| Status | Condition at current n | Display |
| :---: | :---: | :---: |
| Compliant | $\mathrm{n}>\mathrm{n}_{\text {min }}$ <br> and <br> a $\mathrm{n}_{1}$ value with $\mathrm{n}_{\text {min }}<\mathrm{n}_{1}<=\mathrm{n}$ exists such as $\left.\mathrm{pl}\left(\mathrm{n}_{1}\right)\right\rangle=\mathrm{pSCi}$ <br> and $\min \mathrm{p} 2\left[\mathrm{n}_{\text {min }}, \mathrm{n}\right]>\mathrm{pSCi}$ | Probability >p1( $\left.n_{1}\right)$ $\left(\right.$ for $\left.n=n_{1}\right)$ |
| Not compliant | $\begin{gathered} \mathrm{n}>\mathrm{n}_{\min } \\ \text { and } \\ \text { a } \mathrm{n}_{2} \text { value with } \mathrm{n}_{\min }<\mathrm{n}_{2}<=\mathrm{n} \text { exists } \\ \text { such as } \mathrm{p} 2\left(\mathrm{n}_{2}\right)<=\mathrm{pSCi} \\ \text { and } \\ \max \mathrm{p} 1\left[\mathrm{n}_{\text {min }}, \mathrm{n}\right]<\mathrm{pSCi} \\ \hline \end{gathered}$ | Probability < p2( $\mathbf{n}_{2}$ ) (for $\mathrm{n}=\mathbf{n}_{\mathbf{2}}$ ) |
| Not computable | None of the two above conditions | $\begin{aligned} & \text { Probability >p1( } \left.n_{1}^{\prime}\right) \\ & \left(\text { for } n=n_{1}^{\prime}\right) \\ & \text { Probability }<p^{\prime}\left(n_{2}^{\prime}\right) \\ & \left(\text { for } n=n_{2}^{\prime}\right) \end{aligned}$ |

### 4.3. Criteria applicability

The termination criteria and "Protected LEO \& GEO Regions" criteria applicability depends on initial orbit parameters. The following table summarizes the conditions of use for these criteria in STELA software.

|  | LEO | GTO | GEO |
| :---: | :---: | :---: | :---: |
| Termination <br> criterion TC1 | Applicable | Applicable | Applicable |
| Termination <br> criterion TC2 | Applicable | Applicable | Applicable |
|  | Applicable if the <br> initial orbit | Applicable if the <br> initial orbit <br> C1 criterion <br> belongs to protected <br> LEO region | belongs to protected <br> LEO region |
| C2 criterion | Not <br> applicable <br> initial orbit does <br> not belong to protected <br> LEO region | Applicable if the <br> initial orbit does <br> not belong to protected <br> LEO region | applicable |
|  | Not |  |  |
| C3 criterion | Not applicable | Applicable | Not <br> applicable |
| C4 criterion | Not applicable | Applicable | Applicable |

NB: «Applicable» means that the criteria is checked by the software

When using the statistical mode, the termination criteria and "Protected LEO \& GEO Regions" criteria applicability depends on initial/nominal orbit parameters as well. The following table summarizes the conditions of use for these criteria in STELA software.

|  | GTO Statistical Mode |
| :---: | :---: |
| Termination <br> criterion <br> STC 1 | Applicable |
| Termination <br> criterion <br> STC2 | Applicable when automatic stop is selected |
| Termination <br> criterion <br> STC3 | Applicable |
| SC1 criterion | Applicable if the nominal (non dispersed) <br> orbit belongs to protected LEO region |
| SC2 criterion | Applicable if the nominal (non dispersed) <br> orbit does not belong to protected LEO region |
| SC3 criterion | Applicable |
| SC4 criterion | Applicable |

### 4.4. Protected regions criteria status

At the end of the simulation the STELA software checks the compliance with the four "Protected LEO \& GEO Regions" criteria. Five status are possible, and depend on initial parameters and simulation results.

- "Not applicable" : this status is written in the output results when the current "Protected Region" criterion is not applicable (report to §4.3),
- "Not computable" : this status is written in the output results when the current "Protected Region" criterion is fulfilled during the simulation duration, but when the simulation duration is too short to allow any conclusion, or when the number of simulations is too short for a statistical analysis, or when the statistical analysis was not conclusive (report to § 4.2.9) .
- "Not compliant" : this status is written in the output results in the output results when the current "Protected Region" criterion is not fulfilled.
- "Compliant" : this status is written in the output results when the current "Protected Region" criterion is fulfilled during the simulation duration, and when the simulation duration is long enough.
- "Not Reliable" : this status replaces both "Compliant" and "Not Compliant" status for a GTO simulation. It indicates that one single orbit propagation can not give a reliable criterion status due to resonance phenomena (see § 3.9.2 and Warning 2).

Warning 1 : the simulation duration must be:

- at least 25 years in order to allow the STELA software to ensure in any case the compliance with C 1 and C 2 criteria,
- at least 100 years in order to allow the STELA software to ensure in any case the compliance with C2, SC2, C3, SC3, C4 and SC4 criteria.

Warning 2 : for GTO orbits the extrapolation results are very sensitive to the initial conditions (date, perigee position wrt the sun direction...) and to the computation parameters (area, drag and SRP
coefficients, solar activity...) due to the sun-moon perturbation and to resonance phenomena. See §Ref 6. A statistical computation using the "statistical mode" (through GUI or in batch mode) is to be done in order to obtain relevant results.

## 5. STELA fundamentals

Since the first STELA version in 2010, three dynamical models (referred to as LEO, GEO and GTO model) have been developed and tuned for an efficient propagation of LEO, GEO and GTO orbit types. The GTO model is the most generic (because it is able to deal with eccentric orbits), complete and precise one.

## Since STELA V3 (2015) only the previously so-called "GTO dynamical model" remains in STELA software and is now referred to as "STELA dynamical model".

Then whatever simulation the user chooses (LEO, GEO, GTO), the STELA dynamical model will be used to propagate the orbit.
It is used with different settings for the dynamics, depending on the orbit type. For example a $4 \times 4$ earth gravity model can be chosen for a GEO propagation, a 7 x 0 earth gravity model can be chosen for most of the LEO propagations and a $7 \times 7$ earth gravity model can be chosen for the GTO propagations (these are the STELA default settings for these orbit types).

Note also that since STELA V2.6 (2014), STELA uses the CIRF frame as integration frame (previously the integration frame was the Celestial Mean of Date Frame). A complementary acceleration is then considered depending on the choice of the reference frame (can be MOD, ICRF or CIRF, as selected in the advanced parameters files, ICRF being the default value since it is inertial).

### 5.1. Frames

### 5.1.1. Schematic frames transformation



### 5.1.2. Celestial Mean Of Date Frame (MOD)

The Mean Equator and Equinox of Date Frame (O, X, Y, Z) is defined with :

- O the Earth mass center,
- The z-axis aligned with the mean Earth's spin axis,
- The x-axis aligned with the mean equinox of date (vernal direction),
- The y-axis completing the direct orthonormal trihedron.

This frame is not inertial because of the Earth precession.
The user can choose this frame in order to define the initial orbit parameters.

### 5.1.3. ICRF

The ICRF (International Celestial Reference Frame) is defined by the IERS conventions (see $\S$ References 4), and realizes an ideal reference system, by precise equatorial coordinates of extragalactic radio sources observed in Very Long Baseline Interferometry (VLBI) programmes.

The user can choose this frame in order to define the initial orbit parameters.

### 5.1.4. CIRF

The CIRF (Celestial Intermediate Reference Frame) is defined by the IERS conventions. It is deduced from the ICRF by taking into account the precession-nutation model.

This frame is used by STELA for the integration of the spacecraft motion (see §STELA
fundamentals for more information).
The user can choose this frame in order to define the initial orbit parameters.

### 5.1.5. Rotating TIRF

The TIRF (Terrestrial Intermediate Reference Frame) if defined by the IERS conventions as follows :

- O the Earth mass center,
- The z-axis aligned with the true Earth's spin axis at date and north-oriented (it is aligned with the Celestial Intermediate Pole),
- $(x, y)$ : true equator of date,
- The x -axis aligned with the intersection between true equator and Greenwich meridian,
- The y-axis completing the direct orthonormal trihedron.

The TIRF is deduced from the CIRF by a rotation of angle "ERA" (Earth Rotation Angle depending on UT1) around $z$-axis. It can also be deduced from the ITRF (International Terrestrial Reference Frame) by rotations describing the «polar motion».

The user can choose this frame in order to define the initial orbit parameters.

### 5.1.6. Terrestrial Frozen at Epoch

The Terrestrial Frozen at Epoch frame (TFE) is a TIRF-like frame, but frozen at a reference date, and whose x -axis is oriented along a reference longitude. It can be deduced from the CIRF (at reference date) by two consecutive rotations along the z -axis :

- First rotation of angle "ERA"
- Second rotation of angle "reference longitude"

The user can choose this frame in order to define the initial orbit parameters.
Note that when using this frame in the statistics mode and dispersing the initial date, the difference frozen epoch - initial date is kept.

### 5.1.7. EME2000

The "Earth Mean Equator and Equinox at epoch J2000" frame is very close to ICRF (the difference is about 0.02 arc seconds). Indeed the transformation is the following:

$$
\begin{aligned}
& \vec{X}_{I C R F}=B \vec{X}_{\text {EME } 2000} \\
& \vec{V}_{I C R F}=B \vec{V}_{\text {PME2000 }}
\end{aligned}
$$

With:

$$
\begin{gathered}
B=R_{3}(-\gamma) R_{1}\left(-\varphi_{3}\right) R_{3}\left(\psi_{b}\right) R_{1}\left(\varepsilon_{a}\right) \\
\left\{\begin{array}{l}
\gamma=-0.052928^{\prime \prime} \\
\varphi_{b}=84381.412819^{\prime \prime} \\
\psi_{b}=-0.041775^{\prime \prime} \\
\varepsilon_{a}=84381.406^{\prime \prime}
\end{array}\right. \\
\boldsymbol{R}_{\mathbf{l}}(\theta)=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{array}\right] \quad \boldsymbol{R}_{2}(\theta)=\left[\begin{array}{ccc}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{array}\right] \quad R_{3}(\theta)=\left[\begin{array}{ccc}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{array}\right]
\end{gathered}
$$

The user can choose this frame in order to define the initial orbit parameters.

### 5.1.8. Transformations from ICRF to MOD

The transformation from ICRF to the Celestial Mean of Date Frame is a two step transformation:

1. From ICRF to EME2000 using a constant frame bias (see above),
2. From EME2000 to MOD using Lieske precession theory.

### 5.1.9. Transformations from CIRF to MOD

The transformation from CIRF to the Celestial Mean of Date Frame is a multi-step transformation:

1. From CIRF to ICRF by taking into account the precession-nutation model,
2. From ICRF to MOD as explained above.

As this process can take an important amount of time, a simplified conversion is also available. This simplified conversion is used during the computation of Sun and Moon positions (raw outputs are in MOD frame and need to be converted to CIRF frame). These positions are computed many times as they are needed for drag computation, sun and moon perturbations, short periods computation, so the use of a simplified conversion is necessary to speed up extrapolations.

### 5.1.10. Local orbital frame (T, N, W)

The local orbital frame $(\mathrm{O}, \mathrm{T}, \mathrm{N}, \mathrm{W})$ is defined with :

- O the spacecraft center of mass,
- T the axis along the track (parallel to the spacecraft velocity),
- W the axis along the orbit kinetic momentum,
- N the axis completing the direct orthonormal trihedron.

This frame is used in STELA software to compute the atmospheric drag force.

### 5.1.11. TEME

The True Equator Mean Equinox frame (TEME) is the frame in which TLE are displayed.
It is a CIRF-like, frame with the X -axis pointing toward the vernal direction defined by Greenwich Mean Sideral Time 1982.
It can be deduced from the PEF (Pseudo Earth Fixed) frame (at a reference date) by a Z-axis rotation of GMST angle, see $\S$ Ref 10.
PEF can be deduced from TIRF by a Z-axis rotation of angle s' ( $0.00005^{\prime \prime}$ ).

This frame is used in STELA software to import TLE data.

### 5.2. Orbital Elements

The term "nature" is related to mean/osculating and the word "type" is used to describe the orbit parameters form (cartesian, keplerian, ...)

### 5.2.1 Orbit parameters nature

The Osculating orbit corresponds to the real spacecraft state vector (position and velocity) at a given date.
The Mean orbit corresponds to the osculating orbit minus the "short period" variations ("short period" < orbit period).

In STELA, the mean orbit parameters are those computed by the semi-analytical model at each integration time step, as the osculating parameters at a given date are the mean parameters plus the added short period effects, computed in the integration frame.

The perturbations taken into account in the short period computation depend on the perturbations defined for the integration. See Osculating orbit § 5.4.2

### 5.2.2 Orbit parameters type

Different types of orbit parameters are used in STELA :

- Type 0 : parameters ( $\mathrm{Zp}, \mathrm{Za}, \mathrm{i}, \Omega, \omega, \mathrm{M}$ ) where :
- " Zp " is the perigee geocentric altitude : $\mathrm{Zp}=\mathrm{a}$.(1-e) $-6,378 \mathrm{~km}$
- "Za" is the apogee geocentric altitude : $\mathrm{Za}=\mathrm{a} .(1+\mathrm{e})-6,378 \mathrm{~km}$
- " i " is the inclination
- " $\Omega$ " is the right ascension of the ascending node
- " $\omega$ " is the argument of periapsis
- " M " is the mean anomaly.
- Type 1 : Cartesian parameters (position and velocity)

$$
(\mathrm{x}, \mathrm{y}, \mathrm{z}, \dot{x}, \dot{y}, \dot{z})
$$

- Type 2 : Keplerian parameters (a, e, i, $\Omega, \omega, \mathrm{M}$ )
- Type 3 : Near-circular elements (a, ex, ey, i, $\Omega, \mathrm{M}+\omega$ ) where
- ex $=$ e. $\cos (\omega)$ is the first component of the eccentricity vector
- ey $=e \cdot \sin (\omega)$ is the second component of the eccentricity vector
- Type 6 : Parameters for non singular eccentricity :

$$
\left(L=\sqrt{\mu a}, e \cos (\omega), G=L \sqrt{1-e^{2}} \cos (i), \omega+M, e \sin (\omega), \Omega\right)
$$

- Type 7 : Poincaré parameters for non singular eccentricity and inclination :

$$
\left(\mathrm{X}_{1}=\sqrt{2 P} \sin p ; \mathrm{Y}_{1}=\sqrt{2 P} \cos p ; \mathrm{X}_{2}=\sqrt{2 Q} \sin q ; \mathrm{Y}_{2}=\sqrt{2 Q} \cos q ; \mathrm{L}=\sqrt{\mu \mathrm{a}} ; \lambda_{\mathrm{eq}}=\Omega+\omega+\mathrm{M}\right) \text { with : }
$$

$$
\begin{gathered}
\eta=\sqrt{1-e 2} \\
\mathrm{p}=-\omega-\Omega \\
\mathrm{q}=-\Omega \\
\mathrm{P}=\mathrm{L}-\mathrm{G}=\mathrm{L}-\mathrm{L} \eta \\
\mathrm{Q}=\mathrm{G}-\mathrm{H}=\mathrm{L} \eta-\mathrm{L} \eta \cos (\mathrm{i})
\end{gathered}
$$

- Type 8 : Equinoctial and near circular elements (a, ex, ey, ix, iy, $\Omega+\omega+\mathrm{M}$ ) where :
- ex $=e \cdot \cos (\omega+\Omega)$ is the first component of the eccentricity vector
- ey $=e \cdot \sin (\omega+\Omega)$ is the second component of the eccentricity vector
- ix $=\sin (\mathrm{i} / 2) \cdot \cos (\Omega)$ is the first component of the inclination vector
- iy $=\sin (\mathrm{i} / 2) \cdot \sin (\Omega)$ is the second component of the inclination vector
- $\chi=\omega+\Omega+M$

The use of orbit parameters types is the following:

- Differential equation: type 8 ,
- Transformations mean <=> osculating: type 8 ,
- Atmospheric drag force: type 1,
- Input / output orbit parameters: type 0 , type 1 , type 2 , type 8

Note that the type 6 parameters have a singularity for null inclination.
Similarly, the type 8 parameters have a singularity for an inclination of $180^{\circ}$, so has the STELA dynamic model.

### 5.3. Time scales

### 5.3.1. TAI

The International Atomic Time is a time coordinate based on the readings of approximately 150 atomic clocks around the world, each corrected for known environmental and relativistic effects. A few clocks, such as the cesium clock ensemble at the U. S. Naval Observatory, carry considerable weight in the TAI.

### 5.3.2. UT1

The Universal Time (UT1) is a measure of the actual rotation of the Earth. It is the observed rotation of the Earth with respect to the mean sun corrected for the observer's longitude with respect to the Greenwich Meridian and for the observer's small shift in longitude due to polar motion.

### 5.3.3. UTC

By definition, the Coordinated Universal Time (UTC) and TAI have the same rate. But some adjustments are regularly made so that the UT1 deviates from UTC no more than $\pm 0.9$ seconds.

$$
\begin{gathered}
\mid \text { UTC }- \text { UT1 } \mid<0.9 \text { seconds } \\
\text { UTC }=\text { TAI }- \text { (number of leap seconds) }
\end{gathered}
$$

The Terrestrial Dynamic Time (TT or TDT) is a theoretical time, which is tied to TAI by a constant offset of 32.184 seconds. Initially, the TT was used to replace the old Ephemeris Time model at the beginning of 1984. Thus, the Terrestrial Dynamic Time runs parallel to UTC :

$$
\mathrm{TT}=\mathrm{TAI}+32.184 \text { seconds }=\mathrm{UTC}+(\text { number of leap seconds })+32.184 \text { seconds }
$$

### 5.3.5. TDB

The Barycentric Dynamic Time (TDB) is the same as the Terrestrial Dynamic Time (TT) except that the TDB takes into account the relativistic corrections due to the Earth's motion in the gravitational potential of the solar system. These corrections amount to as much as about 1.6 milliseconds and are periodic with an average of zero. The dominant terms in these corrections have annual and semi-annual periods.
Planetary motions are now computed using Barycentric Dynamic Time (TDB), which is more uniform than TT.

### 5.3.6. Assumptions for STELA

Since STELA runs long-term extrapolations in the future, and because future leaps on UTC can not be predicted, the STELA software generates propagations assuming that:

- The Barycentric Dynamic Time and the Terrestrial Dynamic Time are approximately equal: TT = TDB
- The difference TT-UT1 remains constant in STELA processings. The default value is:

TT_MINUS_UT1 $=$ TT - UT1 $=($ TT - TAI $)+($ TAI - UTC $)+($ UTC - UT1 $)=32.184+36+0=$ 68.184 seconds (Values at 01/09/2015). TT_MINUS_UT1 is used to compute the date in TT used: in Lieske and al. theory of precession to compute frame conversion EME2000 / Mean Of Date, to compute the frame conversion ICRF/TIRF and also to compute the Sun and Moon positions using the simplified Meeus \& Brown theory. TT MINUS UT1 default value can be modified by the user through the Advanced Parameters panel.

The date of orbital parameters are expressed in the Universal Time UT1.
NB: a 0 UTC-UT1 value can be generally considered. The real value is significant especially in precise frame conversions from or to TIRF or when importing TLE.

### 5.3.7. CNES Julian day

Opposite to classical Julian days (JD), which are counted from January 1st of 4713 BC at midday, CNES 1950 Julian days (JD1950) start is January 1st of 1950 AC at midnight.

JD1950 $=$ JD - 2433282.5

### 5.3.8. Modified Julian Day (MJD)

The Modified Julian Day is defined as:
MJD $=\mathrm{JD}-2400000.5$
The MJD is a downward rounded JD (Julian Date) that would start on 00:00 November 17, 1858.
In STELA, MJD are given as fractional days:
MJD $=$ MJDN + SEC

Where MJDN is the integer part of the Modified Julian Day and SEC its decimal part in seconds between 0 (midnight) and 86400 (next day).

### 5.4. Propagation model

### 5.4.1. Mean orbit

Aimed at long-term simulations, STELA software is based on a semi-analytic extrapolator method. The short periods have been removed from the evolution of orbital elements, allowing a large save of computation time without losing precision on long term (several years) mean evolution.

If $E_{n}^{\text {mean }}$ represents the mean orbital parameters state at the date $\mathrm{t}_{\mathrm{n}}$, and $E_{n+1}^{\text {mean }}$ the mean orbital parameters state at the date $\mathrm{t}_{\mathrm{n}+1}$, then the state $E_{n+1}^{\text {mean }}$ is deduced from the state $E_{n}^{\text {mean }}$ with the use of the derivative $\frac{d E^{\text {mem }}}{\mathrm{dt}}\left(\mathrm{t}_{\mathrm{n}}\right)$. This derivative is calculated through the perturbation forces as follows:
$d E_{\text {Kepler }}^{\text {meal }}$
dt defines the non-perturbed Kepler movement of the spacecraft around the Earth (representing the gravitational force between two points).

represents perturbations due to the Earth potential irregularities (the J2 contribution due to the Earth oblateness, ...).
$\frac{d E_{\text {huisoler_potential }}^{\text {mag }}}{d t}$
represents perturbations due to the gravitational forces of the Moon and the Sun.
$\frac{d E_{\text {crag }}^{\text {mean }}}{d t}$
dt represents perturbations due to the atmospheric drag.
dt represents perturbations due to the solar radiation pressure.

The short periods have been analytically removed from the expression of the perturbations above so that only the middle and long term evolution of the orbital parameters are integrated. The integrator is numerical and is based by default on a sixth-order Runge-Kutta method (the classical fourth-order Runge-Kutta method can be chosen instead in the advanced parameters file located in the configuration folder). The combination of those two methods explains why the propagator is said to be "semi-analytic".

The types of perturbations which can be accounted for in the propagation are described in the following table. See the corresponding paragraphs for more information.

| Perturbation | STELA Model |
| :---: | :---: |
| Earth's gravity field | Recurrence formulation for Zonal terms. Generic formulation for Tesseral terms. $\mathrm{J}^{2}{ }^{2}$ included |
| Solar and Lunar gravity | Yes |
| Atmospheric drag | Yes <br> (Oblate Earth, Rotating Atmosphere, Simpson Quadrature...) |
| Solar radiation pressure (SRP) | Yes (Simpson Quadrature...) |
| Earth's shadow for SRP | Yes* (The Earth shadow is a cylinder) |
| Solid tides | orders 2 and 3 <br> (Sun and Moon contribution) |
| Numerical Integrator | Sixth-order Runge Kutta** |

*: the Earth shadow can be disabled through the "stela_advanced_parameters" file.
**: a Fourth-order Runge-Kutta can be chosen through the "stela_advanced_parameters" file.
Note : since STELA V2.6 (2014) the dynamical model can also consider a complementary acceleration to take into account the fact that its integration frame (CIRF) is non-inertial. The complementary acceleration depends on the choice of the reference frame (can be MOD, ICRF or CIRF, as selected in the advanced parameters files, ICRF being the default value since it is inertial).

STELA default settings depending on the orbit type:

| Perturbation | LEO <br> simulation | GEO <br> simulation | GTO <br> simulation |
| :---: | :---: | :---: | :---: |
| Earth's gravity <br> field | 7x0 including <br> $\mathrm{J}^{2}$ | 4x4 including <br> J2 $^{2}$ | 7x7 including <br> J2² $^{2}$ |
| Solar and Lunar <br> gravity | Yes | Yes | Yes |
| Atmospheric drag | Yes | No | Yes |
| Solar radiation <br> pressure (SRP) | Yes | Yes | Yes |
| Solid tides | No | No | No |
| Numerical <br> Integrator | Sixth-order <br> Runge Kutta** | Sixth-order <br> Runge Kutta** | Sixth-order <br> Runge Kutta** |

### 5.4.2. Osculating orbit

The osculating parameters are computed in the integration frame as follows for each orbit parameter (n):

$$
\mathrm{E}_{\mathrm{n}}^{\text {osc }}=\mathrm{E}_{\mathrm{n}}{ }^{\text {mean }}+\text { short_period }{ }_{\mathrm{n}}
$$

The type of perturbations taken into account in the short period computation depends on the enabled forces.

| Perturbation | Short period Model |
| :---: | :---: |
| Earth's gravity field | $\mathrm{J}_{2}{ }^{*}$ |
| Solar and Lunar gravity | yes $^{* *}$ |
| Atmospheric drag | yes*** $^{\text {Solar radiation pressure (SRP) }}$ |
| yes**** |  |

* If Zonal perturbations are disabled from the equations of the mean movement, then the J2 contribution in the short period computation is disabled as well.
** If Sun and Moon perturbations are disabled from the equations of the mean movement, then their respective contributions in the short period computation are disabled as well.
*** If Drag perturbation is disabled from the equations of the mean movement, then its contribution in the short period computation is disabled as well. Drag short periods are only used for osculating ephemeris writing and standalone mean - osculating conversions. They are not used when computing criteria.
**** If SRP perturbation is disabled from the equations of the mean movement, then its contribution in the short period computation is disabled as well.


### 5.4.3. Partial derivatives

A STELA feature is the ability to compute the partial derivatives of the orbital parameters. Partial derivatives can be used for covariance matrix propagation or in an orbit determination process. STELA propagates the orbital elements and their partial derivatives at the same time using a semi-analytical method, allowing a large save of computation time without losing precision on long term mean evolution.

Let us introduce the vector $\sigma=\left\lfloor E_{0} K_{1} K_{2}\right\rfloor$ with $\mathrm{E}_{0}$ being the initial state vector, $\mathrm{K}_{1}$ a multiplying factor of the drag force and $K_{2}$ a multiplying factor of solar radiation pressure. Then the partial derivatives are
$\frac{\partial E}{\partial \sigma}$.
$\partial \sigma$. The type of perturbation taken into account for the propagation model of the solve-for vector is derived from the dynamical model. It uses non singular orbital elements (type 8, see §5.2.2) and is valid for high eccentricity and high inclination orbits.

| Perturbation | Partial derivatives |
| :---: | :---: |
| Earth's gravity field | $\mathrm{J} 2, \mathrm{~J} 2^{2}, \mathrm{~J} 3, \mathrm{~J} 4, \mathrm{~J} 5, \mathrm{~J} 6, \mathrm{~J} 7$ |
| Solar and Lunar <br> gravity | Yes |
| Atmospheric drag | Yes <br> (Oblate Earth, Rotating Atmosphere, Simpson <br> Quadrature...) |
| Solar radiation pressure <br> (SRP) | Yes |
| Earth's shadow for SRP | No |
| Numerical Integrator | Sixth-order Runge Kutta |

To compute the partial derivatives make sure that the flag "transitionMatrix" is set to true in the STELA advanced parameters file (stela_advanced_parameters.properties in "configuration" folder).

Then, the partial derivatives will be computed and saved in the state transition matrix file (see §A.7. State transition matrix file) when saving the simulation.

The force model or recomputation time step used in the propagation of the partial derivatives can be modified in the STELA advanced parameters file.

### 5.5. Algorithm features for STELA model

Note that the following describes the algorithms for a single extrapolation. The results given by the single extrapolations are then used to perform the statistical analysis via Monte-Carlo methods for GTO (see §5.7).

### 5.5.1. Earth potential

The derivatives of mean parameters due to Earth potential zonal perturbation are analytically expressed using a recurrence formula with the J 2 up to J 99 contributions at first order (however, numerical issues due to cancellations may occur for degrees higher than J 25 . As a consequence it is not recommended to go above this degree), and J2 contribution at second order ( $\mathrm{J}^{2}$ ). Tesseral terms of the Earth potential are also taken into account leading to a complete Earth potential model: each effect of the tesseral harmonics terms which as a period greater than a tunable value expressed as a multiple of the integrator step is included (§Ref.12).

### 5.5.2. Lunisolar potential

The lunisolar potential computation is based on the knowledge of the Sun and Moon positions that are computed using a simplified Meeus and Brown model. The Meeus and Brown model used in STELA has a 6 terms development in longitude, 4 terms development in latitude and 4 terms development in Earth/Third body distance.

Then, like for the Earth potential, the lunisolar perturbation is developed in Poisson series. These series are developed to the order 8 (but as a default value only the development up to order 4 is used; this can be tuned through the "stela_advanced_parameters" file).

Note that Meeus and Brown model originally outputs a position in MOD frame, that has to be converted to integration frame. To decrease computation time, a simplified conversion is used (see §Frames).

### 5.5.3. Atmospheric drag force

In Stela software, the atmosphere is supposed to rotate at the same velocity as the Earth (rotating atmosphere). The oblate shape of the Earth is taken into account. No wind is considered. Therefore, the atmospheric drag force can be easily expressed in the integration frame at any date as :

$$
\vec{F}=-\frac{1}{2} \rho \frac{\mathrm{SCd}}{m} V \vec{V}
$$

where :

- Cd is the drag coefficient
- $S$ is the cross sectional area representing the spacecraft
- is the atmosphere density
- $\vec{V}$ is the satellite velocity with regard to the rotating Earth
- $m$ is the satellite total mass


### 5.5.3.1. Drag Coefficient

The drag coefficient can be defined

- as constant with a value chosen by the user
- variable vs altitude being read in the file "stela_drag_coefficient" (see §Appendix A.1)
- computed by the Cook formula

The file of n Cd values is used in the following way, h being the geodesic spacecraft altitude and i being a line numbering the file $(1<\mathrm{i}<\mathrm{n})$ :

- if $\mathrm{h}(\mathrm{i})<\mathrm{h}<\mathrm{h}(\mathrm{i}+1)$ then $\mathrm{Cd}(\mathrm{h})=\mathrm{Cd}(\mathrm{h}(\mathrm{i}))$
- if $h>h(n)$ then $\mathrm{Cd}(\mathrm{h})=\mathrm{Cd}(\mathrm{h}(\mathrm{n}))$
- if $h<h(1)$ then $\operatorname{Cd}(h)=h(1)$


## Cook formulae:

The $\mathrm{C}_{\mathrm{d}}$ is computed in line with the mean cross sectional area hypothesis. It is based on the value of the drag coefficient of a plate in tumbling mode (§Ref. $3 \& 5$ ):

$$
\begin{aligned}
& C_{d m e a n}=C_{d}^{a}+C_{d}^{\gamma} \quad \text { with: } \\
& C_{d}^{a}=2\left(1+\frac{1}{s^{2}}-\frac{1}{4 s^{4}}\right) \text { erf }(s)+\frac{2 s^{2}+1}{\sqrt{\pi} s^{3}} \exp \left(-s^{2}\right) \\
& C_{d}^{\gamma}=\frac{\sqrt{\pi}}{3 s}\left(2 \sqrt{\frac{T_{w}}{T}}+\sqrt{1-\alpha}\left(s+1-2 \sqrt{\frac{T_{w}}{T}}+(s-1)\left[\operatorname{erf}(s)+\frac{1+\left(2 s^{2}-1\right) \exp \left(-s^{2}\right)}{2 \sqrt{\pi} s^{3}}\right]\right)\right) \\
& \qquad \alpha=\frac{k \mu}{(1+\mu)^{2}}, \quad \text { with } \quad \mu=\frac{M}{M O} \leq 1 \\
& \qquad s=\frac{V}{v_{m}} \quad \text { with } \quad v_{m}=\sqrt{2 r T} \quad \text { and } \quad r=\frac{\Im}{M} \\
& \qquad \text { erf }(x)=\frac{2}{\sqrt{\pi}} \int_{0}^{x} \exp \left(-\xi^{2}\right) d \xi \\
& C_{d}^{a}: \text { absorption coeffcient } \\
& C_{d}^{\gamma}: \text { re- emission coeffcient } \\
& V: \text { plate velocity } v s \text { atmosphere } \\
& T: \text { temperature of the gas (atmosphere) } \\
& T_{w}: \text { wall temperature of the plate } \\
& \left.\Re: \text { perfect gas constante (8.314 } 472 \mathrm{~J} \cdot \mathrm{~mol}{ }^{-1} \cdot \mathrm{~K}^{-1}\right) \\
& M: \text { mean molar mass of the gas } \\
& M O \text { : molar mass of oxygen atom }\left(16.10^{-3} \mathrm{~kg}\right) \\
& k: \text { accomodation constant, from } 2 \text { to } 4, \text { recommanded value from } 3.6 \text { to } 4
\end{aligned}
$$

$\mathrm{V}, \mathrm{T}$ and M are computed by STELA. $\mathrm{T}_{\mathrm{w}}$ and k are tunable in the "stela_advanced_parameters" file:

- $\mathrm{T}_{\mathrm{w}}$ (not very sensitive, the higher this value the higher the $\mathrm{C}_{\mathrm{d}}$ ): default value $=300^{\circ} \mathrm{K}$,
- k (sensitive, the higher this constant the lower the $\mathrm{C}_{\mathrm{d}}$ ): default value $=4$.


### 5.5.3.2. Mean area

The mean area is the area $S$ to be used for drag computation, that is to say the cross sectional area perpendicular to the velocity direction. The user can use the STELA Mean Surface Area tool (see §Tools) to compute it.

This area is constant during the simulation.

### 5.5.3.3. Atmospheric density

There are three atmospheric models available in STELA:

- The empirical model "NRLMSISE-00" ("NRL" for US Naval Research Laboratory, "MSIS" for Mass Spectrometer and Incoherent Scatter radar, "E" for the model that extends from Earth ground through exosphere, and " 00 " for the year of release). The calculation of density needs the knowledge of the date, the satellite position, the Sun position, and data on solar and geomagnetic activities.
The NRLMSISE-00 model implemented in STELA is adapted from the C implementation available on the following Internet site : http://www.brodo.de/english/pub/nrlmsise/index.html

Note that the model implemented by Stela uses double precision and has a more precise Pi value than the reference one.

- The "US-76" model. This model is simpler than the NRLMSISE-00 and does not take into account the variations of the solar activity.
- The Jacchia 77 model. This model is simpler and faster than the NRLMSISE-00 while keeping a good accuracy. The calculation of density needs the knowledge of the date, satellite position, Sun position and data on solar and geomagnetic activities.


### 5.5.3.4. Solar Activity

The solar activity is defined by the geomagnetic activity Ap and the solar flux F10.7. The solar activity can be :

- constant and tuned by the user
- variable vs time being read in the file "stela_solar_activity " (see Appendix A.2.)


### 5.5.3.5. Atmospheric bounds computation

In order to save computation time, an upper atmospheric boundary $\mathrm{Z}_{\text {atmo }}$ is used in the following way:

- If the orbit perigee is higher than $\mathrm{Z}_{\mathrm{atmo}}$, no atmospheric drag is computed.
- If the orbit apogee is lower than $\mathrm{Z}_{\text {atmo }}$, the atmospheric drag is computed on the entire orbit.
- If the other cases, "input" and "output" anomalies (Ve and Vs) are computed and used in the quadrature process:

$\boldsymbol{V}_{e}=-\cos ^{-1}\left(\frac{1}{e}\left(\frac{a\left(1-e^{2}\right)}{Z_{\mathrm{atmo}}+R_{e q}}-1\right)\right)$

$$
v_{s}=-V_{e}
$$

$\mathrm{z}_{\text {atmos }}$ is tunable is the "stela_advanced_parameters" file ("configuration files" directory)

### 5.5.3.6. Simpson's quadrature

The atmospheric drag effect on osculating parameters can be easily computed, but we rather need to know the effect on mean parameters. The STELA software uses the Simpson quadrature method to compute the drag perturbation effect on mean parameters.

First, the drag perturbation is computed at "Nquad" osculating points as follows:

- The mean orbital parameters are propagated from the Ve anomaly to the Vs anomaly of the orbit at the (Nquad-2) points (points are equidistant in true anomaly in order to have more points near the perigee) : these are the quadrature points
- The mean parameters of quadrature points are converted to osculating parameters
- At each quadrature point, using osculating parameters :
- the atmospheric drag force is computed
- the atmospheric drag force is transformed in the (T, N, W) frame
- the derivatives of the osculating parameters $\frac{d E_{\text {dag }}^{\text {osc }}}{d t}$ are computed using the drag force in the (T, N, W) frame and the Gauss formula

Then the drag perturbation on mean parameters is computed using the Simpson quadrature :

$$
\frac{d E_{d r a g}^{m e a n}}{d t}=\frac{1}{2 \pi} \int_{V e}^{V_{v}} \frac{d E_{d r a g}^{a s c}}{d t}=\sum_{1}^{\mathrm{Nquad}} \frac{\left(1-e_{i} \cos E_{i}\right)^{2}}{\sqrt{1-e_{i}^{2}}} \frac{d E_{d a g}^{o s c}}{d t}
$$

Note : the sum is done following the Simpson theory, the first term using the eccentricity and the eccentric anomaly takes into account the repartition in true anomaly of the quadrature points.

### 5.5.4. Solar radiation pressure

The solar radiation pressure force is defined as follows :
For low Earth positions, the solar radiation pressure may have a significant influence on the long term orbit evolution for particular orbits that lead to a phasing between the J2 drift effect and the SRP effect (resonance)

$$
\overrightarrow{F p}=C_{R} P_{0} S\left(\frac{d_{0}}{d}\right)^{2} \vec{u}
$$

Where :

- The albedo of the Earth is not taken into account
- Cr is the reflectivity coefficient
- P0 is the solar constant at 1 UA (see §Physical parameter values )
- $S$ is the reflecting area representing the spacecraft
- d0 $=1$ UA (see §Physical parameter values )
- d is the sun/spacecraft distance
- $u$ is the sun/spacecraft vector
- The reflectivity coefficient is a constant value given by the user at the GUI. It should be greater than 1 (absorbent surface) and less than 2 (reflecting surface)
- S is the cross sectional area perpendicular to the sun/spacecraft direction. The user can use the Stela Mean Surface area tool (see § Tools)
- The sun/spacecraft vector is compute using the simplified Meus \& Brown model.

The solar radiation pressure is recomputed at every integration sub-step.
In order to get an appropriate estimation of the perturbation that takes into account the eclipse duration and its position on the orbit, a Simpson quadrature is used. The process is described below :

- Computation of eccentric anomalies at the entry $\left(\mathrm{E}_{\mathrm{in}}\right)$ and exit $\left(\mathrm{E}_{\text {out }}\right)$ of the eclipse (Earths shadow is considered as a cylinder)
- Determination of " $M$ " quadrature points evenly spaced in eccentric anomaly, between $E_{\text {out }}$ and $E_{\text {in }}$ (lighted up part of the orbit)
- Computation of the solar radiation pressure perturbation in the inertial frame
- Expression of this perturbation at each quadrature point in the "TNW" orbital frame
- Derivatives computation with Gauss equations
- Simpson quadrature (alike the one for the atmospheric drag)

Note that it is possible to deactivate the eclipses through an advanced parameter in "stela_advanced_parameters" file. The eclipse model is then conservative and does not imply any Simpson quadrature, which is faster.

### 5.5.5. Solid tides

Solid tides can be computed up to the degree 3, taking into account the contribution of the sun and the moon. The model relies on the same kind of equations as for the Earth zonal perturbation, but with different inputs.
In library mode the user can choose to deactivate the sun or moon contribution. The values of Love coefficients used by STELA are given in the stela_physical_parameters file

### 5.5.6. Complementary acceleration

In STELA the integration frame is the CIRF frame, which is not inertial. Depending on the considered reference frame, a complementary acceleration has to be taken into account. The reference frame can be MOD, ICRF or CIRF, as selected in the advanced parameters files, ICRF being the default value since it is inertial.

The complementary acceleration is computed using the following equation:

$$
\vec{a}=-2 \vec{\Omega}_{R / R 0} \wedge \vec{V}_{/ R}-\vec{\Omega}_{R / R 0} \wedge\left(\vec{\Omega}_{R / R 0} \wedge \vec{r}\right)-\frac{d}{d t}\left(\vec{\Omega}_{R / R 0}\right) \wedge \vec{r}
$$

Where :

- Omega(R/R0) : angular velocity of frame R relatively to frame R0
- $\mathrm{V}(/ \mathrm{R})$ : velocity of the spacecraft in frame R
- r : position of the spacecraft

The derivative of $\mathrm{Omega}(\mathrm{R} / \mathrm{R} 0)$ is computed using Gauss equations.
Computation is handled at every integration step, by a Simpson quadrature method (but points are equally distributed regarding their eccentric anomaly and not their mean anomaly).

### 5.5.7. $180^{\circ}$ inclination singularity

The type of orbital parameters used in STELA semi-analytic theory leads to a singularity when $\mathrm{i}=180^{\circ}$.
Then the mean inclination value is clamped to $179.5^{\circ}$ when $\mathrm{i}>179.5^{\circ}$.

### 5.6. Iterative mode for LEO and GEO orbits

### 5.6.1. Iterative research of a specific Low Earth Orbit

STELA software is able to work in a "LEO Iterative mode". This computation mode allows the user to determine an initial orbit that will have an expected lifetime given by the user in the GUI. Two iteration modes can be chosen :

- The "eccentric orbit" computation : the STELA software will look for an initial orbit with the same apoapsis altitude than the one defined by the user in the GUI.
- The degree of freedom is the periapsis altitude Zp .
- The other initial parameters ( $\mathrm{Za}, \mathrm{i}, \Omega, \omega, \mathrm{M}$ ) are not modified.
- The "frozen orbit" computation : the STELA software will look for an initial orbit with frozen eccentricity.
- The degree of freedom is the initial semi-major axis.
- The eccentricity is computed as a function of semi-major axis, inclination and Earth potential development ( $\mathrm{k}=7$ ie up to J15).

$$
\begin{aligned}
e_{G}=\frac{2 \sum_{K=1}^{\infty} K \cdot J_{2 K+1}\left(\frac{a_{e}}{a}\right)^{2 K+1} \sum_{l=0}^{K} D_{l}^{2 K+1}(\sin i)^{2 K+1-2 l}}{J_{2}\left(\frac{a_{e}}{a}\right)^{2}\left(3-\frac{15}{4} \sin ^{2} i\right)} \\
D_{m}^{n}=\frac{[2 n-2 m]!(-1)^{m}}{m!(n-m)!\left[\frac{1}{2}(n-1)-m\right]!\left[\frac{1}{2}(n+1)-m\right]!2^{2 n-2 m}}
\end{aligned}
$$

- The eccentricity and argument of Periapsis are defined as follows:

$$
\left\{\begin{array}{c}
e=e_{g} \\
\omega=\frac{\pi}{2} \cdot \operatorname{sign}\left(e_{g}\right)
\end{array}\right.
$$

- The other parameters $(\mathrm{i}, \Omega, \mathrm{M})$ remain as defined by the user in the GUI.

Note that:

- The default expected lifetime for iterative mode is 24.75 years, as a margin to handle little lifetime sensibility to the start of simulation date due to Luni Solar perturbation.
- If the initial orbit has a lifetime smaller than the expected one the iterative mode stops after the first extrapolation.
- The precision on the expected lifetime can be given by the user in the GUI (through the "Algorithm convergence threshold" field in the advanced parameters view). Default value is 10 days.
- A maximum simulation duration can be specified by the user in the GUI (through the "Max duration - expected duration" field in the advanced parameters view). If the initial orbit is too high, simulation will stop before the spacecraft has reached the low-limit altitude : it could save computation time. Default value is 75.25 years so for an expected lifetime of 24.75 years STELA will propagate no longer than 100 years. This value can be adjusted for instance for parametric studies.
- STELA uses a zero search function using Brent's method. This function takes either the semi-major axis or the perigee altitude as parameter, depending on the chosen iterative mode, and returns the difference between the actual and the expected lifetime. Initial bounds for function convergence are: [reentry altitude ; initial altitude] (plus Earth radius, depending on the chosen iterative mode).
- If a resonance phenomenon between the J 2 and the Solar Radiation Pressure occurs, the function could be non-monotonic. As a consequence, the algorithm could become non-convergent.
- In the vicinity of the critical inclination ( $\mathrm{i}_{\mathrm{crit}^{ \pm}} \pm$) the orbit is naturally frozen; the frozen eccentricity $\left(e_{\mathrm{g}}\right)$ cannot be computed and becomes irrelevant. The eccentricity and the argument of Periapsis used are the one defined by the user in the initial state.

$$
\left\{\begin{array}{c}
\varepsilon=0.1^{\circ} \\
\sin ^{2}\left(i_{\text {crit }}\right)=\frac{4}{5}
\end{array}\right.
$$

### 5.6.2. Iterative research of a specific GEO Orbit

STELA software is able to work in a "GEO Iterative mode". This computation mode allows the user to search an initial orbit that will stay above a minimal altitude during a given exclusion time, both defined by the user in the GUI.

The degree of freedom is the initial semi-major axis. The initial other parameters (ex, ey, ix, iy, longitude) remain as defined by the user in the GUI. STELA software compute the osculating geocentric perigee altitude at each integration step to evaluate whether it remains above the minimal altitude or not.

Note that:

- The default targeted minimal altitude for iterative mode is $200+\mathrm{hkm}$ above the GEO altitude $35,786 \mathrm{~km} . \mathrm{h}$ is a margin to be considered due to the fact that the modelisation is a simplified modelisation vs reference numerical propagators. Its value is given in § Physical parameters.
- The default exclusion time for iterative mode is 100 years, which is compliant with the GEO region protected criterion
- At the end of an iteration STELA software displays the "current altitude relative to GEO" : it is the minimal altitude (above the GEO altitude $35,786 \mathrm{~km}$ ) that have been reached by the current orbit.
- The precision on the expected minimal altitude can be given by the user in the GUI (through the "Algorithm convergence threshold" field in the advanced parameters view). Default value is 1 km .
- STELA uses a zero search function using Brent's method. This function takes the semi-major axis as parameter and returns the difference between the current minimal altitude and the expected one. Initial bounds for function convergence are: [GEO altitude ; GEO altitude + geoMinMaxDelta]. geoMinMaxDelta is a key parameter (see § Physical parameters for other key parameters) with a default value of 1000 km .


### 5.7. Dispersions used for statistical analysis

Resonances phenomena encountered in GTO region have very strong effects on the orbit evolution and lifetime. A statistical approach is needed to handle these effects and properly estimate GTO evolution (see §Ref 8). The approach selected, the Monte-Carlo method, lies on the principle of dispersing initial / nominal parameters (such as the mass, the orbital elements and so on) and analysing the results in a statistical way (see §4.2.9 for more information on the results analysis).

### 5.7.1. Date/Time Dispersion

The user can disperse the day, the month or the year of the initial date separately or altogether. The dispersion may be Uniform or Gaussian. The uniform dispersion asks for a minimum and a maximum value. The Gaussian dispersion asks for a standard deviation and uses the nominal value entered in the General tab as the mean value.

Hour dispersion follows the same principle.
Note that for a Uniform dispersion, when entering a negative minimum value (or a maximum value greater than 24) it is taken into account as a day change (day before for a negative value and after for a value greater than 24). For a Gaussian dispersion, mean value is the hour entered in the General tab.

Note that if the user wants to obtain a local time of perigee dispersion (because it is a key parameter in the propagation sensitivity to initial conditions), he can give the initial orbit parameters in the Terrestrial

Frozen at Epoch frame and disperse the initial date.
When using the Terrestrial Frozen at Epoch frame and dispersing the initial date, the difference "frozen epoch" - "initial date" is kept constant.

### 5.7.2. Mass, Areas, drag and reflectivity coefficients Dispersions

These parameters can be dispersed in a uniform or Gaussian way:

- The mean or central value is the one filled in by the user in the General tab,
- The standard deviation (for a Gaussian dispersion) is in percent or in the unit of the dispersed parameter
- The Delta (for a Uniform dispersion) is in percent or in the unit of the dispersed parameter

Note that when a variable file is given as a nominal value, the uniform or Gaussian dispersion generates a multiplicative coefficient that is applied to the whole file.

## Correcting the dispersed values:

When dispersing values, non-physical values may appear. These values are corrected following this method:

- Mass : If the generated mass is smaller than $0.1 \%$ of the nominal value, mass is corrected to $0.1 \%$ of the nominal value to avoid value too closed to zero
- Areas : negative dispersed values are corrected to 0
- Reflectivity Coefficient : negative dispersed values are corrected to 0 , and values greater than 2 are corrected to 2
- Drag Coefficient : negative dispersed values are corrected to 0

Note that these corrections change the distribution form that cannot be considered as purely uniform or Gaussian anymore. The user may change the law parameters entered (Standard deviation or Delta) to end up with a real strictly uniform or Gaussian law.

### 5.7.3. Solar Activity Dispersion

### 5.7.3.1. Random Cycles

Random cycles dispersion uses measured values from past solar cycles (F10.7 and Ap) to create a pseudo-real random solar activity:

- Measured values from 1954 are divided into 6 solar cycles (these 6 cycles can be found in the "configuration/Solar_activity_cycles" directory)

- Each solar cycle has its own length (duration)
- A uniform law then generates numbers between 1 and 6 to create a solar cycle sequence long enough to match the extrapolation duration
- A uniform law generates a departure point in the first cycle

Note that the user can add its own solar cycles files in the folder "configuration/Solar_Activity_Cycles". Stela will recognize them given that the format is respected. Number of files and length of these files are automatically detected by Stela.

### 5.7.3.2. Uniform / Gaussian dispersion

Uniform and Gaussian dispersion of the Flux F10.7 and Ap follow the method described in §5.7.2

### 5.7.3.3. Mixed (3 date ranges)

When "Mixed" solar activity is selected, the solar activity file is used to generate solar activity data with no dispersion from the beginning date of simulation until "date 1" and with uniform or Gaussian dispersion from date 1 to date 2 . After date 2 until the end of simulation a random set of solar cycles (using the measured past solar cycles) is generated. The same approach is applicable to the geomagnetic indices by keeping consistency with the measured solar flux. "Date 1" can be for example the last date of measured data and "Date 2" an expected end date of the current solar cycle. The next figure is an example of a dispersed solar flux with this method.


## Correcting the dispersed values:

When dispersing values, non-physical values may appear. These values are corrected following this method:

- Flux F10.7: negative dispersed values are corrected to 0
- Ap: negative dispersed values are corrected to 0

Note that these corrections change the distribution form, that cannot be considered as purely uniform or Gaussian. The user may change the law parameters entered (Standard deviation or Delta) to end up with a real uniform or Gaussian law.

### 5.7.4. Orbit Parameters Dispersion

Orbit parameters can be dispersed either from a covariance matrix or a correlation matrix, in a uniform or gaussian way.

### 5.7.4.1. Correlation and Covariance Matrices

Considering $\mathrm{X}_{\mathrm{X}}$ as X standard deviation and XY as X and Y covariance, the Covariance Matrix is given by:

$$
\left(\begin{array}{ccc}
\sigma_{X} \sigma_{X} & \sigma_{X Y} & \sigma_{X Z} \\
\sigma_{X Y} & \sigma_{Y} \sigma_{Y} & \sigma_{Y Z} \\
\sigma_{X Z} & \sigma_{Y Z} & \sigma_{Z} \sigma_{Z}
\end{array}\right)
$$

For clarity reasons, a 3 by 3 matrix will be considered here instead of the 6 by 6 used by Stela. The equivalent correlation matrix and standard deviation vector associated will be:

$$
\left\{\left(\begin{array}{ccc}
1 & \frac{\sigma_{X Y}}{\sigma_{X} \sigma_{Y}} & \frac{\sigma_{X Z}}{\sigma_{X} \sigma_{Z}} \\
\frac{\sigma_{X Y}}{\sigma_{X} \sigma_{Y}} & 1 & \frac{\sigma_{Y Z}}{\sigma_{Y} \sigma_{Z}} \\
\frac{\sigma_{X Z}}{\sigma_{X} \sigma_{Z}} & \frac{\sigma_{Y Z}}{\sigma_{Y} \sigma_{Z}} & 1 \\
\sigma=\left(\sigma_{X}\right. & \sigma_{Y} & \sigma_{Z}
\end{array}\right)\right.
$$

These matrices are symmetrical.
Note that when using a Gaussian dispersion, STELA will ask for a Standard deviation vector () but when using a uniform dispersion, Stela will ask for a Delta vector ( $\pm$ ). The link between Standard deviation and Delta is given, for a uniform dispersion, by the relation:

$$
\sigma=\frac{2 \Delta}{\sqrt{12}}
$$

The following formula displays the conversion from one matrix to another:

$$
\begin{aligned}
& \left(\begin{array}{ccc}
\sigma_{X} \sigma_{X} & \sigma_{X Y} & \sigma_{X Z} \\
\sigma_{X Y} & \sigma_{Y} \sigma_{Y} & \sigma_{Y Z} \\
\sigma_{X Z} & \sigma_{Y Z} & \sigma_{Z} \sigma_{Z}
\end{array}\right)=\left(\begin{array}{ccc}
\sigma_{X} & 0 & 0 \\
0 & \sigma_{Y} & 0 \\
0 & 0 & \sigma_{Z}
\end{array}\right)\left(\begin{array}{ccc}
1 & \frac{\sigma_{X Y}}{\sigma_{X} \sigma_{Y}} & \frac{\sigma_{X Z}}{\sigma_{X} \sigma_{Z}} \\
\frac{\sigma_{X Y}}{\sigma_{X} \sigma_{Y}} & 1 & \frac{\sigma_{Y Z}}{\sigma_{Y} \sigma_{Z}} \\
\frac{\sigma_{X Z}}{\sigma_{X} \sigma_{Z}} & \frac{\sigma_{Y Z}}{\sigma_{Y} \sigma_{Z}} & 1
\end{array}\right)\left(\begin{array}{ccc}
\sigma_{X} & 0 & 0 \\
0 & \sigma_{Y} & 0 \\
0 & 0 & \sigma_{Z}
\end{array}\right) \\
& \left(\begin{array}{ccc}
1 & \frac{\sigma_{X Y}}{\sigma_{X} \sigma_{Y}} & \frac{\sigma_{X Z}}{\sigma_{X} \sigma_{Z}} \\
\frac{\sigma_{X Y}}{\sigma_{X} \sigma_{Y}} & 1 & \frac{\sigma_{Y Z}}{\sigma_{Y} \sigma_{Z}} \\
\frac{\sigma_{X Z}}{\sigma_{X} \sigma_{Z}} & \frac{\sigma_{Y Z}}{\sigma_{Y} \sigma_{Z}} & 1
\end{array}\right)=\left(\begin{array}{ccc}
\frac{1}{\sigma_{X}} & 0 & 0 \\
0 & \frac{1}{\sigma_{Y}} & 0 \\
0 & 0 & \frac{1}{\sigma_{Z}}
\end{array}\right)\left(\begin{array}{ccc}
\sigma_{X} \sigma_{X} & \sigma_{X Y} & \sigma_{X Z} \\
\sigma_{X Y} & \sigma_{Y} \sigma_{Y} & \sigma_{Y Z} \\
\sigma_{X Z} & \sigma_{Y Z} & \sigma_{Z} \sigma_{Z}
\end{array}\right)\left(\begin{array}{ccc}
\frac{1}{\sigma_{X}} & 0 & 0 \\
0 & \frac{1}{\sigma_{Y}} & 0 \\
0 & 0 & \frac{1}{\sigma_{Z}}
\end{array}\right)
\end{aligned}
$$

Note that when the standard deviation of a parameter is null, covariances and correlations with this very parameter are null as well.

Dispersion through these matrices uses a Cholesky decomposition.

## Correcting the dispersed values:

When dispersing values, non-physical values may appear. These values are corrected following this method:

- Eccentricity: negative dispersed values are corrected to 0 , dispersed values greater than 1 are corrected to $1-\varepsilon$,
- Inclination : when $\mathrm{i}<0, \mathrm{i}=-\mathrm{i}$ and $\Omega=\Omega+180^{\circ}$, when i is greater than $180^{\circ}$ the usual correction is applied.

Note that these corrections change the distribution form, that cannot be considered as purely uniform or Gaussian. The user may change the law parameters entered (Standard deviation or Delta) to end up with a real uniform or Gaussian law.

### 5.8. Physical and key parameter values

Here are some physical parameters values used in STELA :

- Earth potential model GRIM 5 including
- Harmonics up to order and degree 99
- Earth radius
- Earth flattening
- Standard gravitational parameter

These parameters are saved in the "stela _earth_potential_coefficient" file in the "configuration files" directory.

- Geocentric Earth radius for criteria verification and "ha" and "hp" computation : 6,378 km
- Astronomical unit : $1 \mathrm{UA}=1.49598022291 \mathrm{E} 11 \mathrm{~m}$
- Solar Radiation Pressure at 1UA : 0.45605E-5 N/m2
- Sun standard gravitational parameter : 1.32712440018E20 m3.s-2.kg-1
- Moon standard gravitational parameter $4.9027779 \mathrm{E} 12 \mathrm{~m} 3 . \mathrm{s}-2 . \mathrm{kg}-1$

These parameters are saved in the "stela_physical_parameters" file in the "configuration files" directory.

Here are other key parameters not to be modified (saved in the "stela_internal_parameters.properties" file in the "resources" directory):

- GEO margin used in C4 criterion verification: $\mathrm{h}=3 \mathrm{~km}$
- LEO margin used in C2 criterion verification: $\mathrm{h}=2 \mathrm{~km}$
- GTO margin used in C2, C3 and C4 criteria verification: $\mathrm{h}=10 \mathrm{~km}$
- ${ }^{\text {max }}$ used in C3 criterion verification $=2$ years
- GEO limit inclination $=15^{\circ}$


### 5.9. Validity domain

The following paragraphs describe STELA validity domain. The validity domain for each kind of simulation (LEO, GEO, GTO) corresponds to the validation domain.

### 5.9.1. STELA validation

STELA results have been validated for a certain range of parameters. To ensure that this range is respected, parameters are tested at extrapolation start, this control of validity domain is described in §5.9.2.

STELA validation has consisted in a comparison with runs of CNES reference numerical propagators (PSIMU and ZOOM) including complete dynamical models of forces. The STELA precision is about $1 \%$ on a computed lifetime of 25 years and better than 2 km for the minimum and maximum altitudes for 100 years for LEO extrapolation. The precision is better than 3 km for the minimum and maximum altitudes for 100 years GEO extrapolation.

For GTO extrapolations, the precision is better than 10 km for the minimum and maximum altitudes for 100 years extrapolation except for orbits close to "critical inclinations". For lifetime computation the result of one run may not be very close to a numerical propagator result but statistical results based on several runs are. This is due to resonance phenomena between earth gravity field and sun-moon perturbation. See Ref 6 and Ref 8.

### 5.9.2. Control of validity domain

Before extrapolating, each parameter is controlled and compared to "authorized" and "recommended" (warning limits) intervals. If value is out of the authorized interval, the extrapolation doesn't start, and a message appears in the logbook to indicate the necessary correction.

## - General blocking limits

| Params | Interval | Explanation |
| :---: | :---: | :---: |
| Perigee altitude <br> (Type 0) | $] 0 ;+\inf [$ | Physics coherence |
| Apogee altitude <br> (Type 0) | $[\mathrm{Zp} ;+\inf [$ | Physics coherence |
| Eccentricity | $[0 ; 1[$ | Physics coherence |
| Mass | $] 0 ;+\inf [$ | Physics coherence |
| Reflecting Mean |  |  |


| area | ] 0 ; +inf [ | Physics coherence |
| :---: | :---: | :---: |
| Reflectivity coefficient | [ $0 ;+\inf$ [ | Physics coherence, $\mathrm{C}_{\mathrm{r}}=0$ means that the SRP is not taken into account |
| Simulation duration | [ $0 ;+\inf$ [ | Physics coherence |
| Integration step | ] 0 ; + inf [ | Physics coherence |
| Date of the initial orbit | All existing dates | Physics coherence |
| Reference date of the "Terrestrial Frozen at Epoch" Frame | All existing dates | Physics coherence |
| Ephemeris step | ] 0 ; + inf [ | Math coherence |
| Constant solar activity : Flux et Ap | [ 0 ; + inf [ | Physics coherence |
| Drag Mean area | ] 0 ; + inf [ | Physics coherence |
| Drag coefficient | [ 0 ; + inf [ | If null value, simulation is executed with no drag force |
| Number of points of Simpson drag quadrature | $\begin{gathered} {[3 ;+\inf [-} \\ 2 \mathrm{P} \end{gathered}$ | Number must be uneven, this is required by the theory. |
| Period of drag reevaluation | [ $1 ;+\inf$ [ | Expressed as a number of integration step |
| Order of Fourier series in drag short periods computation | [ $0 ;+\inf$ [ | Order must be a positive integer |
| Number of points of Simpson solar radiation pressure quadrature | $\begin{gathered} {[3 ;+\inf [-} \\ 2 \mathrm{P} \end{gathered}$ | Number must be uneven, this is required by the theory. |
| Order of Fourier series in SRP short periods computation | [ $0 ;+\inf$ [ | Order must be a positive integer |
| Reentry altitude | [ 0; + inf [ | Physics coherence |
| Earth zonal potential order (without recurrence formulation) | [0; 15] | Not implemented beyond J15 |
| Earth zonal potential order (with recurrence formulation) | [0; 99] | Order 99 is max order provided by GRIM 5 STELA file |


| Earth tesseral potential order | [0; 95] | Degree 95 is max degree provided by GRIM 5 STELA file |
| :---: | :---: | :---: |
| Tesseral minimum period | [ 0 ; + inf [ | Physics coherence |

- Blocking limits for LEO simulations

| Params | Interval | Explanation |
| :---: | :---: | :---: |
| Expected lifetime <br> (only in iterative <br> mode) | $] 0 ;+\inf [$ | Physics coherence |
| Max duration - <br> expected duration <br> (only in iterative <br> mode) | $[0 ;+\inf [$ | Coherence |
| Algorithm <br> convergence <br> threshold (only in <br> iterative mode) | $] 0 ;$ Expected |  |
| lifetime [ | Coherence |  |

- Blocking limits for GEO simulations

| Params | Interval | Explanation |
| :---: | :---: | :---: |
| Exclusion <br> duration (only in <br> iterative mode) | $10 ;+\inf [$ | Physics coherence |
| Min. perigee <br> altitude minus <br> GEO altitude <br> (only in iterative <br> mode) | $[0 ;+\inf [$ | Coherence |
| Algorithm <br> convergence <br> threshold (only in <br> iterative mode) | $] 0 ;+\inf [$ | Coherence |

- Blocking limits for Statistical Analysis

| Params | Interval | Explanation |
| :---: | :---: | :---: |
| Mass |  |  |
| Uniform <br> dispersion $(\mathrm{kg})$ | $[0 ;+\inf [$ | Physics coherence |
| Gaussian <br> dispersion $(\mathrm{kg})$ | $[0 ;+\inf [$ | Physics coherence |
| SRP |  |  |
| Area |  |  |
| Uniform |  |  |


| dispersion (m²) | [ $0 ;+\inf$ [ | Physics coherence |
| :---: | :---: | :---: |
| Gaussian dispersion ( $\mathrm{m}^{2}$ ) | [ 0 ; + inf [ | Physics coherence |
| Coefficient Cr |  |  |
| Uniform dispersion (\%) | [ 0 ; + inf [ | Physics coherence |
| Gaussian dispersion (\%) | [ $0 ;+\inf$ [ | Physics coherence |
| Drag |  |  |
| Area |  |  |
| Uniform dispersion $\left(\mathrm{m}^{2}\right)$ | [ 0 ; + inf [ | Physics coherence |
| Gaussian dispersion ( $\mathrm{m}^{2}$ ) | [ $0 ;+\inf$ [ | Physics coherence |
| Constant coefficient Cd |  |  |
| Uniform dispersion (\%) | [ $0 ;+\inf$ [ | Physics coherence |
| Gaussian dispersion (\%) | [ 0 ; + inf [ | Physics coherence |
| Variable <br> coefficient or <br> Cook coefficient <br> Cd |  |  |
| Min Uniform dispersion (\%) | [ -100 ; + inf [ | Physics coherence |
| Max Uniform dispersion (\%) | [ -100 ; + inf [ | Physics coherence |
| Gaussian dispersion (\%) | [ $0 ;+\inf$ [ | Physics coherence |
| Solar activity |  |  |
| Variable (file) |  |  |
| Flux F10.7 |  |  |
| Min Uniform dispersion (\%) | [ -100; + inf [ | Physics coherence |
| Max Uniform dispersion (\%) | [ -100; + inf [ | Physics coherence |
| Gaussian dispersion (\%) | [ $0 ;+\inf$ [ | Physics coherence |
| Ap |  |  |
| Min Uniform dispersion (\%) | [ -100; + inf [ | Physics coherence |
| Max Uniform dispersion (\%) | [ -100 ; + inf [ | Physics coherence |
| Gaussian dispersion (\%) | [ $0 ;+\inf$ [ | Physics coherence |
| Constant |  |  |
|  |  |  |


| Flux F10.7 |  |  |
| :---: | :---: | :---: |
| Uniform dispersion (sfu) | [ $0 ;+\inf$ [ | Physics coherence |
| Gaussian dispersion (sfu) | [ $0 ;+\inf$ [ | Physics coherence |
| Ap |  |  |
| Uniform dispersion (unitless) | [ 0 ; + inf [ | Physics coherence |
| Gaussian dispersion (unitless) | [ $0 ;+\inf$ [ | Physics coherence |
| Dates | All existing dates | Physics coherence |
| Covariance matrix |  |  |
| XX | [ $0 ;+\inf$ [ | Physics coherence |
| XY | [ - inf ; + inf [ | Physics coherence |
| Correlation matrix |  |  |
| Element | [-1; 1] | Physics coherence |
| Standard deviation | [ $0 ;+\inf$ [ | Physics coherence |
| Delta | [ $0 ;+\inf$ [ | Physics coherence |
| Max number of executions | N* | Physics coherence |
| Number of processors | $\begin{gathered} \hline 10 ; \text { Max } \\ \text { number of } \\ \text { executions } \end{gathered}$ | Physics coherence |

If value isn't in the recommended interval, the degraded extrapolating starts and a message appears in the logbook to describe the warning.

## - General warning limits

| Params | Interval | Explanation |
| :---: | :---: | :---: |
| Maximum <br> computed points <br> by extrapolation | $[0 ; 146200]$ | Computer Coherence |
| Reflectivity <br> coefficient | $[1 ; 2]$ | The reflectivity <br> coefficient should <br> range from 1 to 2 |
| Inclination | $\left[\begin{array}{l}\left.0^{\circ} ; 58.4^{\circ}\right] \mathrm{U} \\ {\left[68.4^{\circ} ; 111.5^{\circ}\right.} \\ ] \mathrm{U}\left[121.5^{\circ} ;\right. \\ 179.5^{\circ}[ \end{array}\right.$ | See below * |
|  |  |  |


| Reentry altitude | $[80 \mathrm{~km} ;+\mathrm{inf}$ |  |
| :---: | :---: | :---: |
| $[$ | Model not validated <br> for lower altitudes |  |
| Integration step | $] 0 ; 24 \mathrm{~h}]$ | Model not validated <br> for larger integration <br> steps |

* Warning limits for inclination has the following source : a computation near the critical inclination value may require to increase the development of the Earth potential for a better accuracy.
- Warning limits for LEO simulations

| Params | Interval | Explanation |
| :---: | :---: | :---: |
| Perigee altitude <br> (Type 0) | [ Reentry altitude ; $2,200 \mathrm{~km}$ ] | LEO validation domain |
| Apogee altitude <br> (Type 0) | $\begin{gathered} {[\mathrm{Zp} ; 2,200} \\ \mathrm{km}] \\ \hline \end{gathered}$ | LEO validation domain |
| Initial Eccentricity | [ $0 ; 0.125$ ] | LEO validation domain |
| Inclination | $\begin{gathered} {\left[0^{\circ} ; 40^{\circ}\right] \mathrm{U}[ } \\ \left.80^{\circ} ; 110^{\circ}\right] \mathrm{U}[ \\ 130^{\circ} ; 180^{\circ} \end{gathered}$ | See below |
| Mean area / mass (reflecting and drag areas) | ] 0 ; 0.1 ] | LEO validation domain |
| Simulation duration | $\begin{gathered} 10 ; 100 \text { years } \\ ] \end{gathered}$ | $\begin{gathered} \text { LEO validation } \\ \text { domain } \end{gathered}$ |
| Reentry altitude | $\begin{aligned} & {[120 \mathrm{~km} ;+} \\ & \inf [ \end{aligned}$ | LEO validation domain |
| Expected lifetime (only in iterative mode) | $\underset{[ }{] 0 ; 100 \text { years }}$ | LEO validation domain |
| Max duration expected duration (only in iterative mode) | $\begin{gathered} {[0 ;} \\ 100 \text { - Expected } \\ \text { lifetime I } \end{gathered}$ | LEO validation domain |
| Inclination (only in iterative mode, frozen orbit) | $\left[\begin{array}{c} {\left[0.5^{\circ} ; 58.4^{\circ}\right]} \\ \mathrm{U} \\ {\left[68.4^{\circ} ; 111.5^{\circ}\right.} \\ ] \mathrm{U} \\ {\left[121.5^{\circ} ;\right.} \\ \left.179.5^{\circ}\right] \end{array}\right.$ | A frozen eccentricity computation near the critical inclination values may not be relevant |

* Warning limits for inclination has the following source: the dynamical properties of LEO inclined between $\left[40^{\circ}, 80^{\circ}\right]$ and $\left[110^{\circ}, 130^{\circ}\right]$. At these inclinations resonance effects due to various perturbation sources (solar radiation pressure, third-body perturbation and drag in particular) have been shown to have significant effects on LEO lifetime in some particular cases. As a result, criteria status may be very sensitive to initial parameters. For more information on the resonance effects please refer to Ref 11.
- Warning limits for GEO simulations

| Params | Interval | Explanation |
| :---: | :---: | :---: |
| Perigee altitude <br> (Type 0) | $[34,786 \mathrm{~km} ;$ <br> $36,786 \mathrm{~km}]$ | GEO validation <br> domain |
| Apogee altitude <br> (Type 0) | $[\mathrm{Zp} ; 36,786$ <br> $\mathrm{km}]$ | GEO validation <br> domain |
| Initial Eccentricity | $[0 ; 0.009]$ | GEO validation <br> domain |
| Initial Inclination | $\left[0 ; 20^{\circ}\right]$ | GEO validation <br> domain |
| Reflecting area / <br> mass | $] 0 ; 0.1]$ | GEO validation <br> domain |
| Simulation <br> duration | $] 0 ; 150$ years |  |
| $]$ | GEO validation <br> domain |  |
| Exclusion <br> duration (only in <br> iterative mode) | $] 0 ; 150$ years |  |
| Min. perigee <br> altitude minus <br> GEO altitude <br> (only in iterative <br> mode) | GEO validation <br> domain |  |
| $0 ; 1,000 \mathrm{~km}$ |  |  |
| Algorithm <br> convergence <br> threshold (only in <br> iterative mode) | GEO validation <br> domain <br> 1,000 km - <br> target altitude <br> $[$ | GEO validation <br> domain |

- Warning limits for GTO simulations

| Params | Interval | Explanation |
| :---: | :---: | :---: |
| Mean area / mass <br> (reflecting and <br> drag areas) | $] 0 ; 0.1]$ | GTO validation <br> domain |
| Simulation <br> duration | d 100 years <br> ] | GTO validation <br> domain |
| Semi-major axis | $[$ Earth radius ; <br> $40900 \mathrm{~km}]$ | GTO validation <br> domain |

## - Warning limits for Statistical Analysis

The _nom parameters are the nominal values entered in the general tab

| Params | Interval | Explanation |
| :---: | :---: | :---: |
| Mass |  |  |
| Uniform <br> dispersion $(\mathrm{kg})$ | [0; Masse_nom <br> $[$ | No negative mass |


| Gaussian dispersion (kg) | $\begin{gathered} {[0 ; \text { Masse_nom }} \\ 13[ \end{gathered}$ | No negative mass |
| :---: | :---: | :---: |
| SRP |  |  |
| Area |  |  |
| Uniform dispersion ( $\mathrm{m}^{2}$ ) | [ 0 ; Sr_nom] | No negative area |
| Gaussian dispersion ( $\mathrm{m}^{2}$ ) | $\left[0 ; \mathrm{Sr}_{\mathrm{f}} \mathrm{nom} / 3\right.$ | No negative area |
| Coefficient Cr |  |  |
| Uniform dispersion (\%) | $[0 ; \min (1-$ <br> $1 / C r$ nom; <br> $2 / C r$ nom -1$) x$ <br> $100]$ | Coefficient between 1 and 2 |
| Gaussian dispersion (\%) | [ $0 ; \min (1-$ 1/Cr_nom; $\begin{gathered} 2 / C r \_ \text {nom - 1) x } \\ 100 / 3] \\ \hline \end{gathered}$ | Coefficient between 1 and 2 |
| Drag |  |  |
| Area |  |  |
| Uniform dispersion ( $\mathrm{m}^{2}$ ) | [ 0 ; Sd_nom] | No negative area |
| Gaussian dispersion ( $\mathrm{m}^{2}$ ) | [ 0 ; Sd_nom / 3] | No negative area |
| Constant coefficient Cd |  |  |
| Uniform dispersion (\%) | [ $0 ; 100$ ] | No negative coefficient |
| Gaussian dispersion (\%) | [ $0 ; 100 / 3$ ] | No negative coefficient |
| Variable coefficient or Cook coefficient Cd |  |  |
| Gaussian dispersion (\%) | [ $0 ; 100 / 3$ ] | No negative coefficient |
| Solar activity |  |  |
| Variable (file) |  |  |
| Flux F10.7 |  |  |
| Gaussian dispersion (\%) | [ $0 ; 100 / 3$ ] | No negative flux |
| Ap |  |  |
| Gaussian dispersion (\%) | [ $0 ; 100 / 3$ ] | No negative Ap |
| Constant |  |  |
| Flux F10.7 |  |  |
| Uniform dispersion (sfu) | $\begin{gathered} {[0 ; \text { F10.7_nom }} \\ ] \end{gathered}$ | No negative flux |


| Gaussian <br> dispersion (sfu) | $[0 ;$ F10.7_nom <br> $/ 3]$ | No negative flux |
| :---: | :---: | :---: |
| Ap |  |  |
| Uniform <br> dispersion <br> (unitless) | $[0 ;$ Ap_nom ] | No negative Ap |
| Gaussian <br> dispersion <br> unitless) | $[0 ;$ Ap_nom / 3 |  |
| [ | No negative Ap |  |
| Max number of <br> executions | $] 0 ; 400000]$ | Average memory <br> limit |
| Max number of <br> processors | Number of <br> physical cores | Optimum use of <br> resources |

- Warning limits for dynamical model

| Params | Interval | Explanation |
| :---: | :---: | :---: |
| Inclination | $\left[0^{\circ} ; 179.5^{\circ}\right]$ | Dynamical model <br> parameters not adapted <br> to inverse equatorial <br> orbits |
| Integration step <br> (inclination above <br> $120^{\circ}$ ) | $10 ; 12 \mathrm{~h}]$ | Recommended <br> integration step |
| Earth zonal <br> potential order <br> (with recurrence <br> formulation) | $[0 ; 25]$ | Accuracy starts to be <br> degraded beyond J25 |

### 5.10. Logbook error list

The following table lists the errors that can appear in the logbook when an exception is thrown.

| Logbook errors and warnings | Signification |
| :---: | :---: |
| Error in validity control of parameters [list] | Listed parameters are out of their expected <br> bounds (blocking limit) and STELA model is <br> not valid outside of. |
| Warning, parameters are out of advocated <br> bounds [list] | Listed parameters are out of their expected <br> bounds (warning limit) and STELA model <br> may not be fully valid outside of. |
| Error initializing solar activity file | Solar activity file or solar cycles files have <br> not been found |
| Error reading solar activity file filename | The solar activity file is corrupted |
| Error initializing variable drag coefficients file | Variable drag coefficient file has not been |
| found |  |$|$| Error reading variable drag coefficients file <br> filename | The variable drag coefficient file is corrupted |
| :---: | :---: |
|  |  |


| The NRLMSISE-00 atmospheric model returns a non-physical value of density, possibly due to bad solar activity inputs | The atmospheric density computation has failed, possibly due to a wrong solar activity file or non-physical orbital parameters |
| :---: | :---: |
| The expected conversion has failed | The conversion of type, bulletin nature or bulletin frame has failed, due to some incoherent input parameters |
| Osculating to Mean bulletin conversion algorithm has not converged | Nature bulletin conversion has failed due to non-convergence of the algorithm. <br> Unfortunately, there is nothing that can be done to overcome it |
| Impossible to plot graph, you don't have enough memory | Plotting data requires memory you don't have. Please decrease the simulation duration |
| Ephemeris file writing failed | The ephemeris file could not be written, possibly due to hardware malfunction or limited disk capacity |
| Out of memory error. Please reduce the extrapolation duration or increase the time step | Memory is full and ephemeris cannot be stored any more. When transition matrix flag is activated, for very long simulations, the required memory can be over 500 MB |
| Extrapolation failed | The propagation has failed, possibly due to wrong internal parameters (if changed) or variable solar activity out of file bounds |
| Iteration failed | The iterative mode has failed, due to the failure of one propagation |
| Inclination very close to critical inclination, the computation keeps the eccentricity vector defined by the user in the initial state | When inclination is very close to critical inclination, eccentricity vector is not adjusted |
| Lifetime of the initial orbit might already be below the expected lifetime, please extrapolate the initial state | It is not necessary to iterate to reduce the lifetime as it is already below the 25 years threshold |
| Error saving file \$ \{ filename \} | The file could not be saved |
| Error loading file \$ \{ filename \} | The file is corrupted |
| Unable to connect to the JMX client on port $n$ | Port $n$ is not available |
| Log from simulation number $n$ : Remote Execution error | Error from process running simulation $n$ |
| Out of memory error. Please reduce maximum number of simulations | All simulations generated for the statistical mode are stored in memory. Requiring too many simulations may lead to a memory overload |
| Statistical analysis failed | The statistical analysis has failed: one of the propagation has failed |
| Non-physical state reached. Try to reduce integration step. | A non-physical state has been detected by STELA, either at the first integration step or during the extrapolation. During the extrapolation a correcting mechanism is used but only for a given number of tries. |
| Fatal error | It should never happen, unless some internal parameters files have been modified |
| Solar cycles random draw is not designed for | The solar cycles random draw algorithm has been designed to ensure independence of Monte-Carlo results wrt simulation duration. |

Recurrence zonal formulation shows numerical issues for orders higher than 25

This is ensured only for simulation duration roughly lower than 10,000 years.
Recurrence zonal formulation uses Legendre polynomials. Beyond J25, accuracy of these polynomial coefficients starts to decrease.

### 5.11. TLE conversion

The conversion of Two Lines Elements to STELA inputs uses the "SGP-SDP4" model in the TEME frame (§Ref 10). The osculating orbital elements are computed from the TLE using SGP4-SDP4 theory. This conversion is not valid for high eccentricities due to limitations in the SGP4-SDP4 short period model. It is then recommended for quasi circular orbits when importing into STELA session. For higher eccentricity orbits, it is recommended to use mean orbital elements when importing into STELA session. The mean elements usable by STELA are directly those of the TLE except for the semi-major axis a" that has to be deduced from the "mean motion". The following equations are used to convert the Kozai-based mean motion n (standard for TLE orbital products) to a Brouwer-based mean motion n " (see Annex B in §Ref 13).

$$
\begin{aligned}
& a_{1}=\left(\frac{\mu}{n}\right)^{2 / 3} \\
& \delta_{1}=\frac{3}{4} \frac{J_{2} \cdot R_{g}^{2}}{a_{1}^{2}} \frac{\left(3 \cos ^{2} i-1\right)}{\left(1-e^{2}\right)^{3 / 2}} \\
& a_{2}=a_{1}\left(1-\frac{1}{3} \delta_{1}-\delta_{1}^{2}-\frac{134}{81} \delta_{1}^{3}\right) \\
& \delta_{0}=\frac{3}{4} \frac{J_{2} \cdot R_{g}^{2}}{a_{2}^{2}} \frac{\left(3 \cos ^{2} i-1\right)}{\left(1-e^{2}\right)^{3 / 2}} \\
& n^{\prime \prime}=\frac{n}{1+\delta_{0}} \\
& a^{\prime \prime}=\left(\frac{\mu}{n^{\prime}}\right)^{2 / 3}
\end{aligned}
$$

The applicable constants are those of WGS-72 (§Ref 10):

- Earth's gravitational constant $=398600.8 \mathrm{~km}^{3} / \mathrm{s}^{2}$
- Earth radius $\mathrm{R}_{\mathrm{E}}=6378.135 \mathrm{~km}$

The proposed ballistic coefficient ( $\mathrm{m} / \mathrm{A} / \mathrm{Cd}$ ) is deduced from $\mathrm{B}^{*}$ (D.Vallado, "Fundamentals of Astrodynamics and Applications", §2.4, p114 et 115) :

$$
B^{*}=\frac{1}{2} \frac{C_{d} S}{m} \rho_{0} R_{T}
$$

With $_{0}=2.461 .10^{-5} \mathrm{~kg} / \mathrm{m}^{2} / \mathrm{E}_{\mathrm{R}}$

Then:

$$
\frac{m}{C_{d} A}=\frac{1}{12.741621 B^{*}} \frac{\mathrm{~kg}}{\mathrm{~m}^{2}}
$$

## 6. Glossary

CCSDS : Consultative Committee for Space Data System
CIRF : Celestial Intermediate Reference Frame
EME2000 : Earth Mean Equator and Equinox at epoch J2000
GEO : Geostationary Earth Orbit
GTO : Geostationary Transfer Orbit
GUI : Graphical User Interface
ICRF : International Celestial Reference Frame
IERS : International Earth Rotation Service
LEO : Low Earth Orbit
MOD : Mean Equator and Equinox of Date
MTCO : Monte-Carlo
SRP : Solar Radiation Pressure
TAI : International Atomic Time
TBC : To Be Confirmed

TBD : To Be Defined

TEME : True Equator Mean Equinox frame
TFE : Terrestrial Frozen at Epoch frame
TIRF : Terrestrial Intermediate Reference Frame
TT : Terrestrial Time
UT1 : Universal Time
UTC : Coordinated Universal Time

## 7. References

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4. IERS Technical note $\mathrm{N}^{\circ} 36$ : IERS CONVENTIONS (2010)
5. Fraysse et al., «Long term orbit propagation techniques developed in the frame of the French Space Act », 22nd ISSFD, 2011
6. Lamy et al., «Analysis of Geostationary Transfer Orbit long term evolution and lifetime», 22nd ISSFD, 2011
7. Le Fevre et al., «Long term orbit propagation techniques developed in the frame of the French Space Act» 5th IAASS Conference, 2011
8. Morand et al., «Dynamical properties of Geostationary Transfer Orbits over long time scales: consequences for mission analysis and lifetime estimation» AIAA-ASC Conference, 2012
9. Le Fevre et al., «Compliance of disposal orbits with the French Space Act : the good practices and the STELA tool»; Acta Astronautica Volume 94, Issue 1, January-February 2014, Pages 234-245 and IAC 2012
10. D.A. Vallado - Revisiting Spacetrack Report \#3: Rev2 - AIAA 2006-6753-Rev2, http://www.celestrack.com/publications/AIAA/2006-6753/
11. Lamy et al., «Resonance Effects on lifetime of Low Earth Orbit Satellites» 23rd ISSFD, 2012
12. Morand, V., Caubet, A. and al., "Semi analytical implementation of tesseral harmonics perturbations for high eccentricity orbits". AAS/AIAA, 2013
13. Felix R.Hoots, History of analytical orbit modeling in the U.S. Space Surveillance System, Journal of Guidance and Dynamics, Vol. 27, No. 2, March-April 2004

## Appendix A.1. Drag coefficient file

The drag coefficient file allows to use a drag coefficient variable vs altitude.
An example of drag coefficient file is presented hereunder. It contains the following values :

- geodetic altitude (km)
- corresponding drag coefficient

The file of n Cd values is used in the following way, h being the geodesic spacecraft altitude and i being a line numbering the file $(1<\mathrm{i}<n)$ :

- if $\mathrm{h}(\mathrm{i})<\mathrm{h}<\mathrm{h}(\mathrm{i}+1)$ then $\mathrm{Cd}(\mathrm{h})=\mathrm{Cd}(\mathrm{h}(\mathrm{i}))$
- if $\mathrm{h}>\mathrm{h}(\mathrm{n})$ then $\mathrm{Cd}(\mathrm{h})=\mathrm{Cd}(\mathrm{h}(\mathrm{n}))$
- if $\mathrm{h}<\mathrm{h}(1)$ then $\mathrm{Cd}(\mathrm{h})=\mathrm{h}(1)$

The user may use its own drag coefficient file by :

- keeping the same format (heading and column)
- replacing the file's name in the stela_elib.properties file (configuration folder)

The drag coefficient values written in the " stela_drag_coefficient " default file are based on the :

- formula described in ref 5 based on ref $1 ., 2$. and 3. (see $\S R e f e r e n c e s)$
- Atmospheric density and molecular composition computed by the empirical model "NRLMSISE-00"

The "stela_drag_coefficient " default file is the recommended one by the French Space Act.
Plot of " stela_drag_coefficient " values :

Sphere or Tumbling Flat Plane Mean Drag Coefficient


## Appendix A.2. Solar activity file

The solar activity file "stela_solar_activity" can be provided into different formats:

- the solar activity file provided by default by STELA
- the solar activity file generated by "Debris Assessment Software".


## 1/ Stela solar activity file:

An example of stela solar activity file is presented hereunder.
It contains the following values:

- Date (JD1950 and seconds)
- daily solar flux F10.7 (sfu)
- eight 3 hr AP index for current day

```
| version 1
# *********************************************************
# STELA SOLAR ACTIVITY FILE
# ******************************************************
# DATE(JJ/1950 + SEC) DAILY_FLUX 8x(3H-AP)
# -----------------------------
255761200 269.17 2 3 3 22 15 7 6 3
255861200 279.7 9 6 9 48 12 9 56 48
255961200 278.7 22 12 9 15 9 5 5 6
2560 61200 287.85 6 7 7 3 4 3 4
256161200 290.944464463
256261200 295.9 0 12 6 9 7 5 6 7
256361200 268.5 7 7 7 12 12 7 5 2
256461200 252 12 18 7 22 18 12 12 15
256561200 236.7 7 7 6 6 18 22 15 22
2566 61200 219.5 18 32 48 27 27 27 32 39
2567 61200 218.4 27 6 27 22 22 4 6 7
256861200 220.5 9 27 7 5 2 2 2 5
256961200 209.3 7 3 5 6 4 4 3 5
257061200 200.2 2 2 6 6 6 4 3 2
257161200 193.7 5 4 5 6 12 9 9 7
257261200 186.6 9 7 7 7 15 5 5 5
257361200 196.775 5 7 7 6 6 5
```

Warning: since STELA v2.5 the mean solar flux parameter is automatically computed by STELA using the daily values, in line with what is expected by the atmospheric model. Hence the corresponding column has been removed. Older files including this parameter are readable by the software but the mean flux is not used. Moreover for the date parameter a new column containing the number of seconds from the beginning of the Julian day has been added. Older files not including this parameter are readable by the software. Then a value of 0 sec is considered.

The user may use its own solar activity file by :

- keeping the same format (heading and column)
- replacing the file's name in "stela_advanced_parameters" file (configuration folder)

The values given in stela_solar_activity file are past measurements (from 1956) and future mean prediction given by Noaa and Nasa. File goes up to year 2458.

STELA - Solar activity File


## 2/ DAS solar activity file:

An example of DAS solar activity file is presented hereunder.
It contains the following value:

- daily solar flux F10.7 (sfu)

```
JD, Solar f10.7 flux (noontime), yyyy mm dd
No missing Days (interpolated)
```

Executed by R L Kelley on $02 / 26 / 2013$

| 2435839.0 | 258.63 | 1956 | 12 | 31 |
| ---: | ---: | ---: | ---: | ---: |
| 2435840.0 | 269.17 | 1957 | 1 | 1 |
| 2435841.0 | 279.70 | 1957 | 1 | 2 |
| 2435842.0 | 278.70 | 1957 | 1 | 3 |
| 2435843.0 | 287.80 | 1957 | 1 | 4 |
| 2435844.0 | 290.90 | 1957 | 1 | 5 |
| 2435845.0 | 295.90 | 1957 | 1 | 6 |
| 2435846.0 | 268.50 | 1957 | 1 | 7 |
| 2435847.0 | 252.00 | 1957 | 1 | 8 |
| 2435848.0 | 236.70 | 1957 | 1 | 9 |
| 2435849.0 | 219.50 | 1957 | 1 | 10 |
| 2435850.0 | 218.40 | 1957 | 1 | 11 |
| 2435851.0 | 220.50 | 1957 | 1 | 12 |
| 2435857. | $7 ค 9.3 ค$ | 1957 | 1 | 13 |

The AP coefficient taken into account is the one contained in the advanced parameters file.

## Appendix A.3. Ephemeris file

Examples of STELA ephemeris file are presented hereunder. It corresponds to an output data file of STELA.

The ephemeris can be saved in two different formats:

- a CCSDS-compliant format called CCSDS_OEM (see "ORBIT DATA MESSAGES", CCSDS 502.0-B-2)

```
CCSDS_OEM_VERS = 2.0
COMMENT CNES - STELA VERSION: 2.6-SNAPSHOT
CREATION_DATE = 2014-10-30T16:44:22.288
ORIGINATOR = CNES
META_START
OBJECT_NAME = EXAMPLE space object
OBJECT_ID = EXMMPLE space object
CENTER_NAME = EARTH
REF_FRMME = CIRF
TIME_SYSTEM = UT1
START_TIME = 2015-08-15T23:00:00.000
STOP_TIME = 2030-07-20T14:41:58.024
META_STOP
COMMENT Nature : Mean
COMMENT Type : Cartesian
COMMENT Format : Date Xl X2 X3 X4 X5 X6
COMMENT Units : km, deg
COMMENT GTO example statistical simulation
```

2015-08-15T23:00:00.000 4111.984385603812 -5108.712989839067 0.0
$7.8652511760983765 \quad 6.3307118816998491 .7802921432886583$
2015-08-16T23:00:00.000 $-13898.148138614548 \quad 34488.561748135646$
$1856.7120442442852-2.1434557455212070 .5542428047605187-0.23533062112721104$
2015-08-17T23:00:00.000 -30341. 49051715354 26821.002499267423
$-1304.9297003776762-0.5679195293374969-1.6808192360159424-0.2610866531543134$
2015-08-18T23:00:00.000 -18144.80752171297 -2417.301488748459
$-2792.4633707965354 .111473714381024-3.1028241925885625 \quad 0.24084476152491355$

- a STELA format called STELA_OEM which uses Modified Julian Days (see 5.3.8.)

```
# STELA_OEM
# COMMENT CNES - STELA VERSION: 2.6-SNAPSHOT
# CREATION_DATE = 2014-10-30T16:44:34.642
# ORIGINATOR = CNES
# META_START
# OBJECT_NAME = EXAMPLE space object
# OBJECT_ID = EXMMPLE space object
# CENTER_NAME = EARTH
# REF_FRAME = CIRF
# TIME_SYSTEM = UT1
# START_TIME = 2015-08-15T23:00:00.000
# STOP_TIME = 2030-07-20T14:41:58.024
# META_STOP
# COMMENT Nature : Mean
# COMMENT Type : Cartesian
# COMMENT Format : MJD sec Xl X2 X3 X4 X5 X6
# COMMENT Units : km, deg
# COMMENT GTO example statistical simulation
57249 82799.99999989523 4111.984385603812 -5108.712989839067 0.0
7.8652511760983765 6.330711881699849 1.7802921432886583
57250 82799.99999989523 -13898.148138614548 34488.561748135646
1856.7120442442852 -2.143455745521207 0.5542428047605187 -0.23533062112721104
57251 82799.99999989523 -30341.49051715354 26821.002499267423
-1304.9297003776762 -0.5679195293374969 -1.6808192360159424 -0.2610866531543134
57252 82799.99999989523 -18144.80752171297 -2417.301488748459
-2792.463370796535 4.111473714381024 -3.1028241925885625 0.24084476152491355
```


## Appendix A.4. Report file

An example of report file is presented hereunder. This file contains output data from STELA.
\#CNES - STELA VERSION: 2.0.0
\#LEO Simulation Report
[ General]
Author: U.N. Owen
Comment : Example LEO simulation file
for STELA
Simulation duration : 100.0 years
Ephemeris step : 864000.0 s
Difference between terrestrial and universal time : 66.184 s
Integration Step : 864000.0 s
Drag quadrature Points : 33
Solar radiation pressure quadrature Points : 11
Atmospheric Drag Recompute step : 2 steps
Solar radiation pressure switch : true
Sun Moon Switch : true
Reentry Altitude : 120.0 km
[ Space Object ]
Mass : 1470.0 kg
Drag Area : $15.0 \mathrm{~m}^{\wedge} 2$
Reflecting Area : $15.0 \mathrm{~m}^{\wedge} 2$
Orbit Type : LEO
Reflectivity Coefficient : 1.5
Drag Coeficent Type : VARIABLE
Name : EXAMPLE space object
[ Atmospheric Model ]
Atmospheric model : NRLMSISE-00
[ Solar Activity ]
Solar Activity Type : MEAN_CONSTANT
AP Constant Equivalent Solar Activity : 15
F10.7 Constant Equivalent Solar Activity : 134.83734638
[ Initial Bulletin ]
Date : 2009-07-29T00:00:00.000
Type : Type2PosVel
Frame : CELESTIAL_MEAN_OF_DATE
Nature: MEAN
a (Semi major axis) : 8562.5 km
e (Eccentricity) : 0.0
I (Inclination) : 98.59 deg
RAAN (Right Ascension of Ascending Node) : 277.51331 deg
w (Argument of perigee) : 0.0 deg
M (Mean anomaly) : 0.0 deg
[ Final Bulletin]
Date : 2109-07-29T00:00:00.000
Type : Type2PosVel

Frame : CELESTIAL_MEAN_OF_DATE
Nature: MEAN
a (Semi major axis) : 8561.84930142 km
e (Eccentricity) : 3.6114658E-4
I (Inclination) : 98.5731535225 deg
RAAN (Right Ascension of Ascending Node) : 178.161920203 deg
w (Argument of perigee) : 157.348667255 deg
M (Mean anomaly) : 37.1931945988 deg
[ Results ]
Effective simulation duration : 100.01 years
Criteria 1 : NotApplicable (Lifetime under 25 years)
Criteria 2 : Compliant (No LEO crossing within 100 years)
Min distance to the LEO protected region $=166.93 \mathrm{~km}$
Criteria 3 : NotApplicable (No GEO crossing between 1 and 100 years)
Criteria 4 : NotApplicable (No GEO crossing within 100 years)

## Appendix A.5. Statistical report file

An example of statistical report file is presented hereunder. This file contains all simulations input/output generated with STELA statistical mode. One line corresponds to one orbit propagation. The number of columns depends on the number and type of dispersed parameters.

Note that:

- Nb is the extrapolation number,
- The first "MJD sec" corresponds to the beginning date of the orbit propagation,
- The second "MJD sec" corresponds to the last date of the orbit propagation,
- Ar is the reflectivity area and Cr is the reflectivity coefficient,
- Ad is the drag area, Cd is the drag coefficient or the multiplying factor of the variable Cd (file),
- F10.7 and Ap are the solar activity coefficients or the multiplying factors of the variable solar activity (file),
- $\mathrm{f}(\mathrm{SCi})$ is the observed probability of SCi criterion for Nb extrapolations (number of $\mathrm{OK} / \mathrm{Nb}$ ), $=-1$ if the criterion is not applicable,
- $\mathrm{p} 1(\mathrm{SCi})$ and $\mathrm{p} 2(\mathrm{SCi})$ are the Wilson confidence interval bounds for Nb extrapolations, $=-1$ if the criterion is not applicable,
- If random cycles, 1 stDay is the day number in the first solar cycle and SeqCycles is an integer that gives the statistical solar cycles sequence: ijklmn corresponds to cycle i then cycle j then cycle k then cycle $m$ then cycle $n(i, j, k, 1, m, n$ from 1 to 9 and 0 for 10 ),
- $\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 3, \mathrm{C} 4$ give the status of $\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 3$ and C 4 criteria: $0=$ not $\mathrm{OK} ", 1=$ "OK", $2=$ "not computable", 3 = "not applicable"

```
# STELA_STAT
# COMMENT Generated by STELA 2.3.1
# CREATION_DATE = 2009-07-29T00:00:00.000
# ORIGINATOR = U.N. Owen
# META_START
# OBJECT_NAME = EXAMPLE space object
# OBJECT_ID = EXAMPLE space object
# CENTER_NAME = EARTH
# REF_FRAME = CELESTIAL_MEAN_OF_DATE
# TIME_SYSTEM = UT1
# META_STOP
# COMMENT Nature : MEAN
# COMMENT Type : Perigee/Apogee
# COMMENT Format : Nb MJD sec Zp Za i raan w M Mass Ad Cd 1stDay SeqCycles C1 C2 C3 C4
MJD sec SimDuration f(SC1) p1(SC1) p2(SC1) f(SC2) p1(SC2) p2(SC2) f(SC3) p1(SC3) p2(SC3)
f(SC4) p1(SC4) p2(SC4)
# COMMENT Units : kg, km, deg, years
# COMMENT Example GTO simulation file
# COMMENT for STELA
```

$5505076331.119000073520 .02705944428600633510 .309988111-1-1-110.30998811100$ 0.69001189
4550410.015035892 .16666662 .92238 .481780505501 .862717722338497413152142130344131 $05505179383.116000122390 .02989400702208413710 .395773031-1-1-110.39577303100$ 0.60422697
5550410.015035892 .16666662 .92238 .481780505502 .236085865783709724383244334420131 $05505016930.8440003078430 .0251771631556263310 .462943981-1-1-110.46294398100$ 0.53705602
6550410.015035892 .16666662 .92238 .481780505502 .095396584231002732341402114143131 $05505152648.211999866180 .02904682903642438410 .516817051-1-1-110.51681705100$ 0.48318295
7550410.015035892 .16666662 .92238 .481780505502 .299209213336369731840141113303131 $0550512085.3279995732010 .027444587928102510 .560933871-1-1-110.56093387100$ 0.43906613
8550410.015035892 .16666662 .92238 .481780505502 .344595436327977223733031442411310 $5504959819.50399973430 .02379837199280200610 .59770331-1-1-110.59770331000 .4022967$ 9550410.015035892 .16666662 .92238 .481780505502 .37622554908476777132310120401310 $5505124525.09799953550 .0281556613303879210 .628809741-1-1-110.62880974100$ 0.37119026
10550410.015035892 .16666662 .92238 .481780505502 .18450944311686572655324203110113 $10550519075.1449999399480 .02766608186300415610 .655462781-1-1-110.65546278100$ 0.34453722

## Appendix A.6. Mean Constant Flux for LEO orbits

The mean constant solar activity is a constant value vs time depending on the ballistic coefficient of the spacecraft and on the initial apoapis altitude of the orbit. It has been tuned, through a statistical approach, to achieve a 25 years re-entry duration as a mean value.

A disposal orbit that re-enters the atmosphere in 25 years with this Mean Constant Solar Activity will have a lifetime that is not modified whether the end of mission date shifts, whereas using a variable solar activity leads to a variability of the computed lifetime :


Considering a 25 years lifetime orbit computed with this value, the real lifetime computed statistically with several past solar cycles and several initial dates in the first cycle would have a mean value of 25 years with a cumulative distribution function as follows:

Orbit Lifetime distribution


This constant equivalent solar activity is computed for LEO orbits at the extrapolation beginning using the formulas:

$$
\left\{\begin{array}{c}
A P=15 \\
F 10.7=194.4+3.17 \log \left(\frac{C_{d} S}{m}\right)-6.86 \log \left(Z_{a}\right)
\end{array}\right.
$$

The following figure plots the F10.7 value computed with this formula for various altitude of apogee and ballistic coefficients :

NB: if Cd has been chosen to be variable, a constant $\mathrm{Cd}=2.2$ value is used in STELA to compute the solar flux.


These solar activity coefficients are used in the atmospheric model in the following way:

- Mean and current solar flux values $=$ F10.7
- 3 H geomagnetic index values $=\mathrm{AP}$

The former 'Mean constant' solar activity coefficients (dating from a fit on previous solar cycles) are recalled here in order to allow old simulations to be run again for non-regression purposes:

$$
\left\{\begin{array}{c}
A P=15 \\
F 10.7=201+3.25 \log \left(\frac{C_{d} S}{m}\right)-7 \log \left(Z_{a}\right)
\end{array}\right.
$$

These coefficients can be changed in stela_internal_parameters.properties resource file (located in 'resources' folder) and correspond to keys 'kZero', 'kUn', 'kDeux' (line 460). STELA must be restarted in order to take new values into account.

The technical compliance with the French Space Operation Act must be demonstrated with the most up-to-date coefficients - the default ones.

## Appendix A.7. State transition matrix file

State transition matrix file ("*_stm.txt") is automatically saved when saving a GTO simulation in which the partial derivatives have been computed (see §5.4.3. Partial derivatives). The transition matrix ephemeris is saved with the same frequency as the bulletin ephemeris. To compute the partial derivatives make sure that :

- the flag "transitionMatrix" is set to true in the STELA advanced parameters file (stela_advanced_parameters.properties in "configuration" folder)
- The state transition matrix file contains the ephemeris file in a STELA_OEM format (see §A.3. Ephemeris file) plus additional columns that are the partial derivatives.

Here is the State transition matrix file header :

```
# STELA_OEM
# COMMENT CNES - STELA VERSION: 2.4.0.4
# CREATION_DATE = 2012-12-07T17:09:12.054
# ORIGINATOR = Default Author Name
# META_START
# OBJECT_NAME = Default Object Name
# OBJECT_ID = Default Object Name
# CENTER_NAME = EARTH
# REF_FRAME = MOD
# TIME_SYSTEM = UT1
# START_TIME = 1998-01-01T00:00:00.000
# STOP_TIME = 1999-01-01T00:00:00.000
# META_STOP
# COMMENT Nature : Mean
# COMMENT Type : Equinoctial
# COMMENT Type : ksi = w + RAAN + M
# COMMENT Format : MJD sec a ex ey ix iy ksi da/daO dex/daO dey/daO dix/daO diy/daO
dksi/daO da/dex0
# COMMENT : dex/dex0 dey/dex0 dix/dex0 diy/dex0 dksi/dex0 da/dey0 dex/dey0 dey/dey0
dix/dey0
# COMMENT : diy/dey0 dksi/dey0 da/dix0 dex/dix0 dey/dix0 dix/dix0 diy/dix0 dksi/dix0
da/diy0
# COMMENT : dex/diy0 dey/diy0 dix/diy0 diy/diy0 dksi/diy0 da/dksi0 dex/dksi0 dey/dksi0
dix/dksi0
# COMMENT : diy/dksi0 dksi/dksi0 da/dK1 dex/dK1 dey/dK1 dix/dK1 diy/dK1 dksi/dK1 da/dK2
dex/dK2
# COMMENT : dey/dK2 dix/dK2 diy/dK2 dksi/dK2
# COMMENT Units : m, rad
# COMMENT This is a default comment.
```


## Appendix B.1. Using STELA as a library

## 1. REQUIREMENTS

In order to use STELA as a Library, you will need:

- An installed version of STELA Software ( that you can download on the following website http://logiciels.cnes.fr/STELA/fr/logiciel.htm )
- A Java editor, Eclipse will be used here as an example.
- STELAs Javadoc (optional, you can download it from the previous link)


## 2. INSTALLATION

In order to use STELA as a Library, you have to take the following steps:

### 2.1. Add the .jar

- In Eclipse, right-click on the project you want to use STELA Library in (along this tutorial, it will be called STELATest).
- Open properties, select Java Build Path, tab Libraries, then click the Add external JARs button
- Go to STELA installation folder (default pathway is: C:\Program FilesISTELA_vX.X.X )
- Open the lib folder
- Select all the .jar (Ctrl + A). If you want initialize message, you must add all module of Stela.
- Remarque: In reality, importing all the JAR files shouldnt be necessary (like junit, jfreechart...). It depends on the way you are going to use the library. Compulsory JARs to use STELA as a library would include: stela-elib, stela-etoo, stela-processing, stela-commons and the commons-math (non exhaustive list).


### 2.2. Set up the environment

STELA needs the resources and the configuration folders in order to work properly. Therefore, your program should start with the following line:

Prop.defineROOT("C:<br>RootDirectory...");
The argument in defineROOT() method being the folder where you installed STELA (it contains bin, configuration, lib and resources sub-folders).

## 3. RUNNING IT

Using STELA library in your Java code is very straight forward. You simply need to call the function you want to use. Open the file "example.java" (e.g. : "C:/Program Files/STELA_v2.0.2 /examples/example.java"). In this example, a new GEO simulation has been created, executed, and then results have been displayed in the Eclipse console.

Remarque : The pathway of the load ("C:/Program Files/STELA_v2.0.2
/examples/example_GEO_sim.xml") is the pathway of the xml file we want to run. If working under Windows environment, it is the direct pathway inverting the slash.

Once the Java class saved, errors may appear; these are due to missing imports declarations. To organise and add automatically imports with Eclipse, press : Ctrl + Shift + o.

In order to know the name of the methods to use and their package, you have to search the Javadoc, presented in the following paragraph.

## 4. JAVADOC

Go and download the Javadoc from the http://logiciels.cnes.fr/STELA/fr/logiciel.htm website. Unzip the folder and then open all the target subfolders and unzip the JAR file that you will find. To access the Javadoc of each STELA module, open the index.html filed stemming from the unzipped JAR.

STELA Javadoc contains the information about STELA code, hence names and exact function of the methods and classes you wish to re-use in your project.

For more information about Javadoc, please visit:
http://docs.oracle.com/javase/1.3/docs/tooldocs/win32/javadoc.html
Here is a brief description of STELA different modules:

- stela-batch : Contains the code of the batch mode (commands, inputs and help sections )
- stela-commons : Basics classes, interfaces, abstract classes (for example, only stela-commons would be necessary to create a new atmospheric model), errors, messages and dates management. Contains the generated code as well (Earth potential, ...).
- stela-eapp : Classes dealing with the GUI (view and controller)
- stela-elib: Classes of STELA physical model: atmospheric model, differential equations, various forces, LOS criteria, nature conversions... It is STELA main component. It also contains Monte-Carlo simulations and probabilities computation
- stela-etoo : Entry point of all simulations.
- stela-processing : Basic routines but non abstract (hence implemented, unlike stela-commons): type and frame conversions, Gauss equations, operations on vectors, etc.
- stela-slib : Only contains classes about the mean area tool.
- stela-tle : Only contains classes about the Two-Line Element tool.
- stela-validation : Validation component.


[^0]:    "Termination criterion TC1 : the extrapolation duration defined by the user has been reached"

